Friction modeling and display in haptic applications involving user performance

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ABSTRACT

We review the state of the art in friction estimation and rendering for haptic interfaces and present a method based on a modified Karnopp friction model. We illustrate some of the advantages of this approach and show how it can be used to create accurate and convincing displays of sliding friction, including pre-sliding displacement and stick-slip behavior. We also present the results of human performance experiments in targeting tasks and show that real and simulated friction produce essentially the same results. In both cases, moderate low-stiction friction can improve performance and high stiction degrades it.

INTRODUCTION

Friction is a ubiquitous element of our daily haptic interaction with the world around us. Virtually every surface that we touch and slide our fingers across and every machine or tool that we operate exhibits friction as an essential part of the experience. Human mechanoreceptors are also exquisitely sensitive to the nuances of friction, including the catch/release phenomenon that accompanies a difference between the static and dynamic coefficients of friction and the vibrations that usually accompany sliding. Even the tiny vibrations that signal the onset of sliding, before gross motion occurs, are detected by the human mechanoreceptors and used in grasp force regulation [Edin et al. 1993].

For realism, a virtual world perceived through a haptic interface should include an accurate representation and display of friction. But given the human sensitivity to the manifestations of friction, this is difficult to achieve. With today's haptic interfaces and simple friction models, users often find that they would rather "turn friction off" than put up with rubbery, jittery simulations.

The technical challenges associated with modeling and displaying friction stem from a couple of sources. First, there is the problem that dry friction is highly nonlinear, posing problems for accurate modeling and display via computer-controlled devices. High bandwidth and servo stiffness are required to accurately reproduce the transition from a rapid buildup of force just prior to sliding to the break-away as sliding commences. Moreover, the largest nonlinearity, i.e., the discontinuity between static and dynamic friction, occurs at essentially zero velocity, when the signal to noise ratio in the sensed velocity is worst. To overcome these difficulties, improved models and methods of displaying friction have been a topic of investigation in the haptics research community. The models draw upon work in controls and robotics for modeling systems with friction and specialize it to the needs of haptic display.

The analysis of mechanical systems with dry, or Coulomb, friction has a rich history going back to the original vibrations work of Helmolz and Rayleigh. The modern treatment stems largely from the work of Den Hartog [1931]. Numerous methods have been proposed for identifying friction coefficients and predicting the response of mechanical systems subject to friction. For recent examples and a review of previous work see [Armstrong, et al. 1994; Haessig and Friedland 1991; Johnson and Lorenz 1992; Majiid and Simaan 1995; Kim et al. 1996; Kim et al. 1997; Liang and Feeny 2001; Constantinescu et al. 2000; Nahvi and Hollerbach 1998].

In much of the controls community, simple friction models are used and the emphasis is on accurately predicting the response of a forced system with friction. For haptic display, this is probably less important than capturing the manifestations of friction to which people are particularly sensitive, including the discontinuity between static and dynamic friction and the amount of pre-sliding displacement that may occur.

For haptic display, Chen et al. [1997] render friction and adhesion in a manner similar to the "bristle" model of friction [Haessig, D. A., Friedland, B., 1991] in which virtual bristles are attached to sliding surfaces. The bristles alternately bond to each other and break away at a rate that depends on the bond strength and compliance. Hayward and Armstrong [2000] demonstrate a variation of the Dahl [1976] model in which friction is a function of...
displacement, as shown in Fig. 1. If we take the spatial derivative of the friction force as

\[
\frac{dF}{dx} = \sigma |1 - F_{s} \text{sgn}(x)/F_{a}| \cdot \text{sgn}(1 - F_{s} \text{sgn}(x)/F_{a})
\]  

(1)

where \(\sigma\) and \(i\) are empirical constants and \(F_{a}\) is the static value of friction (which can be different for motion in the positive and negative directions), we can compute the friction force as

\[
F = \int \left(\frac{dF}{dx} \cdot x\right) \, dt + bx
\]  

(2)

where \(b\) is an added viscous term.

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Fig. 1. The Dahl model for friction rendering is based on displacement and captures prestiding displacement and a hysteresis effect associated with reversal of motion.

An alternative approach is a modified version of the Karnopp [1985] model, which is shown schematically in Fig. 2 and is computed as follows:

\[
F_{\text{friction}}(x, F_{a}) = \begin{cases} 
C_{p} \text{sgn}(x) + b_{p} x & \text{for } x < -\Delta v \\
\max(D_{p}, F_{a}) & \text{for } -\Delta v < x < 0 \\
\min(D_{n}, F_{a}) & \text{for } 0 < x < \Delta v \\
C_{n} \text{sgn}(x) + b_{n} x & \text{for } x > \Delta v 
\end{cases}
\]  

(3)

where \(x\) is the relative velocity between the mating surfaces, \(b_{p}\) and \(b_{n}\) are the positive and negative values of the viscous friction, \(C_{p}\) and \(C_{n}\) are the positive and negative values of the dynamic friction, \(D_{p}\) and \(D_{n}\) are the positive and negative values of the static friction, \(\Delta v\) is the value below which the velocity is considered zero, and \(F_{a}\) is the sum of non-frictional forces applied to the system.

Advantages of the modified Karnopp model include an explicit representation of stiction and treatment of the transition between low velocities and static contact.

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Fig. 2. Modified Karnopp model of friction for haptic display.

**FRICTION IDENTIFICATION PROCEDURE**

Fig. 3 shows the one degree of freedom apparatus used for friction identification and display. (Specifications are given in the Appendix.) For friction identification, the mouse is replaced by a test specimen or device being measured. The motor moves the platform back and forth at low frequencies (0.5-3.0 Hz) with amplitudes of 0.01 meters approximating the motions that humans make when exploring the friction properties of surfaces. The system was allowed to warm up for 10 seconds before taking data. Estimated measurement errors are given in the Appendix.

Identification experiments were performed for a 0.419 Kg aluminum block sliding on hard rubber, teflon, and brass. The mass was treated as an unknown variable for testing the identification procedures.

Solving for parameters in the friction models:

Friction parameters were identified for the modified Dahl and Karnopp models described in the previous section. For the Karnopp model, we first separate the data into two groups, corresponding to positive and negative velocities, respectively. The force balance becomes:

\[
F_{m} = ma + C_{p} \text{sgn}(v_{p}) + b_{p} v_{p} + C_{n} \text{sgn}(v_{n}) + b_{n} v_{n} + \epsilon
\]  

(4)

where \(F_{m}\) is the measured force, \(m\) is the specimen mass, \(C_{p}\) and \(C_{n}\) are the positive and negative static friction coefficients, \(b_{p}\) and \(b_{n}\) are the positive and negative viscous damping coefficients and \(\epsilon\) are errors. The equation can be rewritten as

\[
F_{m} = X\beta + \epsilon
\]  

(5)

to solve for the model parameters by least squares regression. The measured quantities are the accelerations, \(a\) and the velocities, \(v_{p}\) and \(v_{n}\). Given that we have measurement
For the Dahl model, the parameters of the differential equation can be obtained using a nonlinear least squares optimization approach, where the goal is to minimize a function, $f$, subject to parameters, $\beta$:

$$f(\beta) = \sum_{j=1}^{N} (F(\beta, t_j) - F_m(\beta, t_j))^2$$

(7)

$F$ and $F_m$ are the model and measured values of the force, respectively. Similar to the Karnopp model, $\beta$ includes the mass, $m$, the positive and negative friction limits, $D_p$ and $D_n$, and the viscous damping coefficient, $b$. The value $\sigma$ for the stiffness was not included in the estimation as it is quite large for the apparatus and presented numerical problems for the optimization procedures used. With $\sigma$ estimated independently, best results were obtained using the `lsqcurvefit` program from the MATLAB® optimization toolbox. The Levenberg-Marquardt method was used to set the search direction for parameter estimation [Coleman et al. 1999].

**Friction identification results**

Reasonably good fits can be obtained with both the modified Karnopp and Dahl models as shown in Fig. 4. A tabulation of the fitting errors is given in Table 4. in the Appendix. (For additional results see [Richard 2000]). However, the modified Karnopp model was generally easier to use. For the Dahl model, careful selection of $\sigma$ was required to avoid numerical difficulties. The Dahl model worked best for cases such as aluminum sliding on rubber,
with significant pre-sliding displacement and dynamic friction.

As is often the case with identification strategies, accurate measurements of velocity and acceleration are the key to accurate friction and mass estimates. Much of the scatter in Fig. 4. can be attributed to noise in the accelerometer signal. The effect appears smaller for aluminum on rubber, but this is due to the larger magnitude of the friction force, so that inertial forces contribute less to the overall force balance.

In addition to measurement accuracy, the selection of the excitation trajectory is important. While it seems that virtually any input trajectory might suffice, best results were obtained when smooth sinusoidal closed-loop position trajectories were used. The frequency of the input trajectory is limited on the low end by the signal-to-noise ratio and on the high end by the apparatus's bandwidth. Sustained oscillations or other instabilities should not occur.

We are also generally interested in displaying friction in combination with inertia, stiffness and other properties. An effective solution is to create a virtual coupling [Adams and Hannaford 1999; Colgate et al. 1995] with a spring and damper in parallel, interposed between the user and the simulated system. For systems with friction, the virtual coupling provides a couple of particular advantages. Because the friction force is dependent on the motion of a virtual block rather than the motion of the haptic device itself, the position and velocity used to calculate the friction are no longer limited by sensor resolution and sample rate. Also, because the motion of the block is virtual, the block velocity can be set to exactly zero in the "stuck" state.

At each time step we measure the position and velocity of the haptic interface and use them to compute the interaction force applied via the virtual coupling. The friction and other forces on the object are then summed to compute its acceleration for the next time step. The algorithm used to calculate the friction is as follows:

```c
if state = STUCK
  friction = -Kp*(x-x_stuck);
else if abs(friction) > F_static
  state = SLIDING;
  friction = -sign(x-x_stuck)*...
  ...* F_dynamic - B*v;
endif
else
  if abs(v) < DV
    state = STUCK;
    x_stuck = x+sign(ff_old)*(F_static/Kp);
    ff = -Kp*(x-x_stuck);
  else
    ff = -sign(v)*F_dynamic-B*v;
  endif
endif
ff_old = friction;
```

The parameters Kp and DV must be tuned correctly for realistic performance. Kp governs the pre-sliding displacement. If Kp is too low, the device will have moved enough that the velocity exceeds DV before the static friction limit is reached (see Fig. 5). If Kp is too high, instability results. DV is the value of velocity below which the device or specimen is said to be stationary. A larger value of DV increases the number of stick-slip oscillations. A detailed analysis can be found in [Richard 2000].

The virtual coupling must also be tuned to avoid limit cycle oscillations on the one hand and an artificially soft response on the other. For a given system mass, damping and friction and a given sampling rate, Richard [2000] establishes the possible range of the parameters for the virtual coupling. When the coupling is tuned appropriately, the simulated friction can match real friction quite closely. Fig. 6 provides a comparison of real versus simulated friction for a system with a 2.0 N friction force, a 1.0 Kg
Fig. 6. A comparison of simulated versus actual friction for aluminum on teflon (friction force = 2.0 N, mass = 1.0 Kg) with user-supplied motion.

object mass and a sampling rate of 1000 Hz. In both cases, motions are generated by a human subject moving the mouse shown in Fig. 3.

**FRICITION DISPLAY AND HUMAN PERFORMANCE**

Anecdotally, the feel of simulated versus real friction with the system in Fig. 3., tuned according to the guidelines in the previous section, is nearly identical -- especially for combinations such as aluminum on rubber where the friction is high and the friction model matches the measured data closely, as shown in Fig. 4. However, a user challenged to distinguish between real and simulated friction will invariably manage to employ a strategy such as moving very slowly (so that measured velocity resolution is comparatively poor) and applying barely enough force to initiate sliding. Under such conditions, any hardware limitations are exacerbated.

A more meaningful test, therefore, is to see whether simulated friction has the same effects on user performance as real friction in tasks of interest. To this end, we conducted Fitts-type targeting tests [Fitts 1954] on human subjects with varying amounts of real and simulated friction. In a Fitts test, subjects are required to move back and forth between two targets in rapid succession. The index of difficulty is:

\[ I_D = -\log_2\left(\frac{(W - D)}{2A}\right) \]  

where \( A \) is the distance between the targets; \( W \) is the width of the targets and \( D \) is the diameter of the cursor. Our subjects were instructed to move the mouse rapidly back and forth, clicking alternately on red and green targets as they became illuminated in the display in Fig. 7. (A successful mouse click had to be recorded within each target for the next target to illuminate.)

The subjects were 20 right-handed individuals (10 male, 10 female). Each subject completed 45 Fitts tests consisting of 9 different levels of difficulty and five different friction conditions. Before each set of tests, subjects practiced with the apparatus with no additional friction and with real friction produced by attaching an aluminum block sliding on rubber. The ordering of tests was randomized. After each test we recorded the completion time, the number of errors and the average error magnitude. For further details on the testing procedure see [Richard 2000].

Due to large subject-to-subject variability, it was not useful to compute average values of task performance. However, trends across the majority of subjects were evident. To highlight these trends, we plotted data using the template shown in Fig. 8. In each case, the subjects’ performance was compared to their performance for a baseline condition with only the inertia and friction inherent in the apparatus (Table 2). The other conditions included: real friction, produced by aluminum sliding on rubber with approximately 3N of additional resistance; simulated friction tuned to match the real case; simulated high-stiction with the same dynamic coefficient of friction as in the previous cases but a much larger static coefficient of friction; and pure viscous damping with a constant of 35Ns/m, again producing approximately 3N of resistance at the speeds that most subjects employed.

Task completion trends are summarized in Table 1. For the tasks with the highest difficulty (1,4,7) real and simulated friction provided essentially the same results, improving the time performance of most users. Viscous friction produced similar effects. As one might expect, high stiction generally degraded performance, most
Subjects complete the task more quickly with friction, but accuracy is decreased.

Function slows subjects down and reduces accuracy.

Subjects complete the task more slowly, but with greater accuracy.

Subjects complete the task more quickly with friction, but accuracy is decreased.

Fig. 8. User performance is plotted as targeting errors versus time. Trends can be found by identifying the number of subjects in each quadrant under various friction conditions.

noticably in the tasks with the narrowest targets (e.g. 4). Similar results were obtained when comparing the trends in numbers of errors for each subject.

CONCLUSIONS

Various models exist that can effectively model friction. We tested modified Karnopp and Dahl models and obtained good fits with both. However, we preferred the Karnopp model for its ease of implementation and lack of numerical difficulties. For rendering friction a virtual coupling [Adams and Hannaford 1999] gives good results provided that the values of stiffness and damping are carefully chosen. For a given system stiffness, mass, damping and sampling rate, limits on the stiffness of the coupling can be established such that limit-cycle oscillations are avoided while achieving a satisfactorily "crisp" response.

In targeting tests, we were able to render moderate friction accurately enough that users reported that the simulated friction felt realistic when compared to the friction produced by an aluminum plate sliding on rubber. More

Table 1. Summary of completion time results for targeting tasks. The strongest effects are seen for the tasks with the highest index of difficulty (1, 4, 7) where real and simulated friction and viscous damping improve performance. High stiffness degrades concretely, the users' performance was nearly identical for real and simulated friction in Fitts-type targeting tasks.

Going forward, many extensions are possible. More sophisticated models of friction could capture "memory" effects and dwell [Armstrong 1991]. Friction display can should also extend to multiple degrees of freedom, perhaps with different values of friction in orthogonal directions. These kinds of extensions will be necessary for incorporating friction into more general haptic models of surfaces [Loyd and Pai 2001] and mechanisms [MacClean 1996; Dupont et al. 1997].

However, to improve the fidelity of the friction rendering, we would focus first on velocity sensing. Although we used a 2048 line quadrature encoder and a "fixed-distance" or fixed encoder-pulse differentiation scheme (rather than the more common fixed-sample period or fixed time differentiation approach) our results were nonetheless limited by velocity resolution and accuracy.

ACKNOWLEDGEMENTS

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REFERENCES


APPENDIX.

Table 2. Apparatus specifications

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Standard Deviation</th>
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<tbody>
<tr>
<td>System inertia (motor and slide)</td>
<td>0.692 kg</td>
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<td>Force output to commanded voltage</td>
<td>23.315 N/V</td>
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<td>Maximum force (continuous)</td>
<td>9.48 N</td>
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<td>Range of motion</td>
<td>3.2 cm</td>
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<tr>
<td>Position resolution</td>
<td>2.57 x 10^-6 m</td>
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<tr>
<td>Voltage output to measured force</td>
<td>8.896 N/V</td>
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<tr>
<td>Static friction (motor and slide)</td>
<td>0.75 N</td>
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The apparatus shown in Fig. 3. consisted of a low-friction DC servomotor, linear slide and 2048 line quadrature encoder. The servo rate for all experiments was 1000 Hz. System parameters are given in Table 2. The standard deviations of errors when tracking a smooth sinusoid at 2Hz are given in Table 3. These provide an upper-bound estimate of the measurement errors.

Table 3. Standard deviations of the measured errors when tracking a 2Hz sinusoid

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<th>Measurement Error Position</th>
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<tr>
<td>velocity</td>
<td>3.18 x 10^-3 meters/second</td>
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<tr>
<td>acceleration</td>
<td>0.2379 meters/second^2</td>
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<td>force</td>
<td>0.062 Newtons</td>
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Table 4. Friction estimates and standard deviations for two sets of tests involving aluminum on brass, teflon and rubber

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Base1 Estimate</th>
<th>Std Dev</th>
<th>Teflon Estimate</th>
<th>Std Dev</th>
<th>Rubber1 Estimate</th>
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