A MODEL OF THE ATTACHMENT/DETACHMENT CYCLE OF ELECTROSTATIC MICRO ACTUATORS

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ABSTRACT

Several recent electrostatic micromotor designs, including harmonic motors, rely on an attachment/detachment cycle of operation. The cycle commences when the voltage of a particular stator electrode is increased, creating an attractive force between the stator and rotor (which is grounded). This force brings the rotor into contact with the stator (therefore, either the rotor or stator must be insulated). The cycle ends and another begins when the stator electrode is grounded, releasing the rotor, while the voltage of a neighboring stator electrode is increased. Such motor designs exploit the inverse square attraction between image charges to generate large pull-down forces; also, they may incorporate certain geometrical features that result in mechanical advantage. Thus, these motors hold the promise of serving as high-force actuators in the microdomain.

Experience with a linear microactuator based on the attachment/detachment cycle has, however, pointed to a basic difficulty with this type of operation: the accumulation of charge on the interfacial insulation can lead, in the long-term, to "sticking", erratic behavior, and even complete motor failure. A simple model of an attachment/detachment cycle has been used to study the charge accumulation process. Simulations performed with this model indicate that, given a simple commutation scheme for switching stator voltages, resistive-capacitive processes can lead to an accumulation of charge sufficient to cause sticking. Criteria incorporating long-term charging effects are suggested for the design of electrostatic micro motors.

1. Introduction

Commonly, the cost or performance of an electromechanical system is limited by the actuators that power it. For instance, robots based on conventional electromagnetic actuators feature neither the precision nor the speed necessary for advanced automation of semiconductor manufacturing (Busch-Vishniac, et al., 1990); the design of sophisticated microsurgical tools tends to be limited by the sheer size of available actuators (Charles, et al., 1990); and the development of artificial arms that permit natural control and graceful movement is limited by the power-to-weight ratio of state-of-the-art motors (Colgate, 1990). The need for a new generation of actuators that can be used to achieve a higher level of performance in small to medium scale electromechanical (mechatronic) systems has, since the early 1980s, led to substantial enterprise in the development of novel actuation techniques. Among the techniques that have been explored are those based on novel materials, such as shape memory alloys (Dario, et al., 1987, Kuribayashi, 1986), mechatronically active gels (Caldwell and Taylor, 1989), polymeric piezoelectrics (Sessler, 1981), and magnetostrictive alloys (Goodfriend, 1991), as well as those based on novel application of conventional actuators, such as ultrasonic motors (piezoelectric) (Uchino, et al., 1988, Schoenwald, et al., 1988, Kamano, et al., 1987), rubber bladders (pneumatic) (Innega, 1987), and miniature levitated robots (electromagnetic) (Pelrine and Busch-Vishniac, 1987).
Since the mid 1980s, electrostatic micromotors have received an increasing amount of attention, particularly with the advent of silicon micromachining. While electrostatic micromotors have been sporadically investigated for over two centuries, they have never met with success: this is in large part because the energy storage density in an air gap electrostatic field is limited by breakdown to be four orders of magnitude less than the energy storage density in an air gap electromagnetic field (assuming that this is limited by the saturation of the core material). A number of authors, however, have pointed out that physical scaling effects should favor electrostatic actuation at sufficiently small scales (e.g., gap sizes of < 5 μm) (Bollee, 1969, Trimmer and Jelens, 1989, Lang, et al., 1987). This is, in part, due to the Paschen Effect which permits an increase in electrostatic field energy storage density of at least two orders of magnitude for very narrow gaps. Further, electromagnetic actuators may have problems with energy dissipation given very small cross-section coils. Lang, et al., have also pointed out that electrostatic motors are compatible with silicon fabrication, which is probably the most highly developed microfabrication technology available today (Lang, et al., 1987). In 1988, Fan, Tai and Muller (Fan, et al., 1989) demonstrated the first operational silicon machined electrostatic micromotor. Since that time, many more electrostatic micromotors have been demonstrated (Mehregany, et al., 1990, Pister, et al., 1990, Tai, et al., 1989, Tavrow, et al., 1990).

The research reported in this paper is motivated by the potential that electrostatic microactuators have as building blocks for macroactuators, i.e., "artificial muscles." The crucial idea underlying artificial muscle is that the exploitation of electrostatic actuators should include not only physical scaling effects, but also geometric effects. Consider, for instance, the fact that electromagnetic actuators must always include magnetic material to provide a high permeability return path. Electrostatic actuators, on the other hand, do not require return paths, since electric field lines begin and end on charges, which typically reside on surfaces. Consequently, force generating surfaces can be packaged much more densely in an electrostatic actuator. A well designed artificial muscle, composed of closely packed linear electrostatic microactuators, may exhibit a ratio of gap volume to total actuator volume two orders of magnitude greater than a conventional rotary motor.

In summary, compelling reasons exist to develop electrostatic micromotors. The next section reviews a particular class of electrostatic micromotors based on an "attachment/detachment cycle." This class of motors holds significant promise for high force and torque applications in the microdomain. In Section 3, a technical difficulty with these motors, charge accumulation on insulating surfaces, is discussed. In Section 4, a model of an attachment/detachment cycle is introduced and used to simulate the charge accumulation process. Section 5 presents a discussion and conclusions.

2. Attachment/Detachment Operation for High Force Output

Among the electrostatic motor designs developed in recent years, many of the more promising operate on the basis of what may be termed an "attachment/detachment cycle." The best-known example of such a motor is the "harmonic" or "wobble" motor (Trimmer and Jelens, 1989, Jacobsen, et al., 1989), which can be used to illustrate important aspects of the attachment/detachment cycle. Figure 1 is an axial view of a wobble motor consisting of a grounded rotor within a circular cavity formed of insulated stator electrodes. Its operation is very simple. At any point in time, all stator electrodes but one are grounded. The voltage of this electrode is set "high" (typically > 100 V),

![Figure 1: Schematic of a Wobble-type motor exhibiting attachment/detachment operation](image-url)
generating an attractive force between it and the rotor. This force causes the rotor to "attach" to the stator. When the high voltage is "sequenced" to the next stator electrode (in, say, a counterclockwise direction), the rotor is "detached" and forced to reattach to the next stator. Under continuous voltage sequencing, the motion of the rotor may be viewed as a series of attachments to and detachments from individual stators, resulting in a rolling motion around the inside of the cavity. This rolling gives rise to a clockwise rotation about the rotor’s own axis (although this axis is not fixed, thus the dubbing "wobble motor").

A number of other actuator designs that feature attachment/detachment cycles have been investigated. Fujita and Omodaka demonstrated a linear motor reminiscent of a linear roller bearing in which the rollers are actuated in the manner described above (Fujita and Omodaka, 1988). The stator electrodes, instead of forming a cavity, are laid out side by side. Maynard et al. have suggested a trunk-like micromanipulator based on a stack of disks which undergo bistable attachment (Maynard, 1990). In our laboratory, a linear micromotor that employs the attachment/detachment cycle has been developed as a potential building block for artificial muscle (Axland, 1990). The design and operation of this motor are illustrated in Figure 2.

The attachment/detachment operation offers several potential benefits over the more conventional, non-contact operation of a micromotor. Perhaps the most significant is that anti-friction bearings are not required; motor operation depends only on clamp-down and possibly rolling, but not slipping. Indeed, slippage is highly undesirable, and a high coefficient of friction between the rotor and stator is an asset. Another benefit is that attachment exploits the "clamp-down" or "image" force which is quite strong and, in conventional motors, a source of difficulty due to its destabilizing effect.

Finally, some designs, including the wobble and harmonic motors, and the linear motor developed in our laboratory, feature built-in transmission effects. A transmission, of course, is typically needed to adjust the output force or torque of an actuator to a level commensurate with the load. More generally, power transfer is optimized by matching the output impedance of the actuator to the load impedance (Pasch and Seering, 1984). The transmission may be thought of as the means of effecting such a match. In the case of the harmonic motor, the rotor angular velocity, \( \omega \), relates to the angular rate of rolling, \( \Omega \), by:

\[
\omega = \frac{r_r}{r_r - r_s} \Omega
\]  

(1)

Figure 2: Operation of a linear actuator exhibiting the attachment/detachment operation.
where \( r_1 \) and \( r_2 \) are the radii of the rotor and the stator cavity, respectively. In the case of the motor in Figure 2, the rotor speed per unit length, \( \gamma \), is related to the frequency of the attachment/detachment cycle, \( \nu \), by:

\[
\gamma = \frac{(L/\delta)^2}{\nu}
\]  

(2)

while the force per unit cross-sectional area, \( \sigma_L \), is related to the electrostatic normal stress, \( \sigma_e \), by:

\[
\sigma_L = \frac{(L/\delta)^2}{\sigma_e}
\]  

(3)

The geometric parameter, \((L/\delta)^2\), therefore, can be considered a nondimensional gear ratio.

3. A Problem with Attachment/Detachment Operation: Charge Accumulation

Several potential difficulties arise with attachment/detachment operation. For instance, due to contact, these designs must contend with squeeze film effects, which may limit speeds, and with wear. In our experience, the most debilitating problem is the accumulation of charge on interfacial insulation. The linear motor pictured in Figure 2 generally performs well when first turned on, but after some period of operation, the rotor motion becomes sporadic, and eventually stops altogether. After the rotor stops, if it is peeled off the stator and the stator surface is dusted with copy machine toner, an image of the rotor appears. This image indicates a surface charge on the stator insulation, apparently of sufficient magnitude to cause the rotor to "stick." We have also noticed that the number of cycles prior to sticking depends on geometric parameters, such as insulator and gap thicknesses.

The “sticking” problem illustrates a deeper difficulty in electrostatic actuation: while forces are created by charges, control is usually effected by voltages. This poses little difficulty if a simple relation between voltage and charge can be enforced. In general, however, the relationship is complex. The problem of charge accumulation discussed in the sequel illustrates one of the subtleties of the voltage-charge relation. Various other subtleties and associated pitfalls await the electrostatic motor designer are discussed by Price, et al. (Price, et al., 1989).

In order to cope with charge accumulation, the causes must be understood. We have identified three potential causes: contact electrification, breakdown, and charge leakage through the insulator (RC charging).

Contact electrification suggests itself immediately because, ubiquitous in everyday life and a serious hazard in many industrial processes, it has long been studied by generating contacts between dissimilar surfaces. The phenomenon has been well explained for metal-metal contacts in terms of work functions (Harper, 1967). Charging at metal-insulator contacts is not as well understood; the reliability of existing models is questionable when taken beyond their empirical bases. Despite this difficulty, contact electrification is difficult to ignore. Most likely, charge transfer at contact will be an important source of surface charge, unless the contacting materials are identical. Coating both rotor and stator with the same insulation may, in fact, be the wisest approach to coping with this effect.

Given the strong electric fields present in an actuator, breakdown of air also could cause charge accumulation. Various theories account for the stages of breakdown, including corona formation, streamers, and sparks. Although in general the onset and extent of breakdown depend on the electric field strength, many other factors, including surface topology, affect the phenomenon. Indeed, few of the parameters governing theories of breakdown are easily accessible experimentally or are suitable design variables. As with contact electrification, the best means of coping with breakdown may be to avoid it altogether, presumably by using low enough field strengths.

The final mechanism, RC charging, is the simplest to understand, but also possibly the most insidious. An electric field across the insulator causes small but finite currents that in turn produce surface charge. This charge tends to decay according to the RC time constant of the insulator. During attachment/detachment cycling, the processes of accumulation and decay compete, potentially resulting in a complicating effect that, cycle after cycle, increases the surface charge. It is not, a priori, clear how to deal with this problem. In the next section, a model of an attachment/detachment cycle is presented and used to explain the effect of system parameters on the charge accumulation.

Before proceeding, however, there is one additional point that should be addressed. Why does not the contact itself remove surface charge, especially since the contacting rotor is bare, grounded metal? The probable explanation is that the real area of contact—at asperity tips—is far less than the apparent area of contact (Rabinowicz, 1965). Under the normal stresses developed by an electrostatic field, the real area may be less than a percent of apparent. Unless the surface resistivity of the insulator is low (unlikely), this will prohibit contact from removing surface charge.
4. Modeling and Simulation of RC Charging

**Actuator Configuration and Control** — Figure 3 illustrates the "actuator" used in this model. A grounded rotor moves up and down between two grounded stators, which is clearly reminiscent of the linear motor pictured in Figure 2. For simplicity, the problem is taken to be one dimensional; i.e., the rotor does not tilt, and fringing fields are ignored. The motor is assumed to operate in vacuum; the squeeze film effects of air may be added at a later time. The basic operation of the motor is obvious, although many different voltage control schemes are possible. For instance, \( V_1(t) \) and \( V_2(t) \) may be square waves of a fixed frequency, and may be completely or only partially out of phase. We have chosen to investigate a "commutated mode" of operation in which \( V_1 \) switches from high to low and \( V_2 \) switches from low to high upon sensing contact of the rotor with the upper stator, and vice versa for contact with the lower stator. Commutation, because it is asynchronous, permits generation of significant forces over a wide range of load conditions, and is indispensable for mechatronic applications. Our simulation, however, has been performed for only the no-load condition.

Because of the commutation, the meaning of "contact" must be clarified. In the model developed below, intimate rotor/insulator contact would lead to infinite RC time constants; moreover, as motivated above, surface asperities prohibit truly intimate surface contacts except under extreme loads. For these reasons, the rotor is allowed to within only a certain minimum distance of the insulator surface. This minimum separation is considered contact.

The behavior of this actuator model, including charge accumulation, is governed by Gauss' Law, Ohm's Law, and Newton's Second Law. The electrical and mechanical behavior are coupled via Maxwell stresses and capacitance modulation due to the rotor position. These laws and relationships result in a set of state equations that, with initial conditions, completely describe actuator behavior. The derivation of the state equations proceeds by first solving for the electric fields, and then applying Ohm's Law and Newton's Law.

**Electric Field Solution** — The electrostatic potentials in both air gaps (i.e., above and below the rotor) and in both insulators are governed by Laplace's Equation (assuming no space charge):

\[
\frac{\partial^2 \psi_{ij}}{\partial y^2} = 0
\]

(4)

\( \psi_{ij} \) is the potential in either the air gap \( r = g \) or the insulator \( r = i \), above \( j = 1 \) or below \( j = 2 \) the rotor. The solution of this equation is:

\[
\psi_{ij}(y) = -E_{ij}y + C_{ij}
\]

(5)

where \( E_{ij} \) is the electric field strength, and \( C_{ij} \) is a constant of integration. Both \( E_{ij} \) and \( C_{ij} \) must be

---

**Figure 3.** Actuator configuration. The stators are identical, and the rotor has zero thickness.
found from boundary conditions. In the region above the rotor, the boundary conditions are:

\[
0 = -E_g \nabla r + C_g \delta_1 \\
-E_g \nabla \delta + C_g = -E_{11} \nabla \delta + C_{11} \\
e_g E_{11} - e_1 E_{11} = \sigma_1 \\
V_1 = -E_{11}(\delta + t') + C_{11}
\]

- ground potential at rotor surface
- continuity of potential at gap/insulator interface
- Gauss' Law at an interface
- potential at stator electrode surface

The boundary conditions may be used to solve for \(\psi_{g1}(y)\) and \(\psi_{11}(y)\); however, only the electric fields are of interest in the present application. The electric field in the gap dictates the force on the rotor, while the electric field in the insulator dictates the rate of charge accumulation. These fields are:

\[
E_{g1} = \left[ \frac{e_g V_1 + t_1 \sigma_1}{e_g t_1 + e_1 (\delta_g + y_r)} \right] \\
E_{11} = \left[ \frac{e_g V_1 - (\delta_g + y_r) \sigma_1}{e_g t_1 + e_1 (\delta_g + y_r)} \right]
\]

(6) (7)

Similar expressions may be found for the electric field strengths below the rotor.

**State Equations** — With solutions for the electric field strengths available, state equations may be written to describe the temporal evolution of the model. A natural choice of state variables is \(\sigma_1, \sigma_2, y_r,\) and \(v_r\) (velocity of the rotor), corresponding to the capacitive and inertial energy storages in the system. Ohm's Law is the constitutive relation that will be used to relate the rates of change of \(\sigma_1\) and \(\sigma_2\) to the electric field strengths in the two insulators:

\[
\frac{d\sigma_1}{dt} = \frac{1}{\rho_i} E_{g1} = \left[ \frac{e_g}{\rho_i e_g t_1 + \rho_i e_1 (\delta_g + y_r)} \right] V_1 - \left[ \frac{(\delta_g + y_r)}{\rho_i e_g t_1 + \rho_i e_1 (\delta_g + y_r)} \right] \sigma_1
\]

\[
\frac{d\sigma_2}{dt} = \frac{1}{\rho_i} E_{12} = \left[ \frac{e_g}{\rho_i e_g t_1 + \rho_i e_1 (\delta_g + y_r)} \right] V_2 - \left[ \frac{(\delta_g + y_r)}{\rho_i e_g t_1 + \rho_i e_1 (\delta_g + y_r)} \right] \sigma_2
\]

(8) (9)

The rates of charge accumulation and decay are strongly modulated by the rotor position. For instance, the time constant associated with the decay of \(\sigma_1\) is (from Eqn. 8):

\[
\tau_{\sigma_1} = \rho_i e_1 \left( 1 + \frac{e_g}{e_1 (\delta_g + y_r)} \right)
\]

(10)

which is obviously strongly influenced by \(y_r\), reaching a maximum when the rotor contacts the upper stator \((y_r = \delta_g - d)\), and a minimum when the rotor contacts the lower stator \((y_r = -\delta_g + d)\).

In this model, the only forces on the rotor are electrostatic—frictional and restoring forces are ignored. The electrostatic force per unit area (Maxwell stress) on the top and bottom of the rotor is:

\[
F_{up} = \frac{e_g}{2} E_{g1}^2
\]

\[
F_{down} = \frac{e_g}{2} E_{g2}^2
\]

(11) (12)

The state equations for \(y_r\) and \(v_r\) are:

\[
\frac{dy_r}{dt} = v_r; \quad |y_r| \leq \delta_g - d
\]

\[
\frac{dv_r}{dt} = \frac{e_g}{2 \mu_r} \left[ \left( \frac{e_g V_1 + t_1 \sigma_1}{e_g t_1 + e_1 (\delta_g + y_r)} \right)^2 - \left( \frac{e_g V_2 + t_2 \sigma_2}{e_g t_1 + e_1 (\delta_g + y_r)} \right)^2 \right]
\]

(13) (14)

\[
260
\]
Equations 8, 9, 13 and 14 form a complete set of state equations. They are evidently quite nonlinear, and depend on six free parameters. Their nonlinearity suggests that these equations should be investigated via simulation; however, exploring a six dimensional parameter space in this way would not be practical. In order to minimize the number of free parameters, and to make the simulation as robust as possible, the equations will first be normalized.

**Normalization**—The inputs, states, and independent variable (time) are each normalized by a physically meaningful constant to yield a new set of nondimensional variables. The state equations are then written in terms of these variables. For instance, the input voltages, $V_1$ and $V_2$, are normalized by the maximum input voltage, $V_{max}$. The associated nondimensional variables are:

$$
\bar{V}_1 = \frac{V_1}{V_{max}}, \quad \bar{V}_2 = \frac{V_2}{V_{max}}
$$

(15)

Both $\sigma_1$ and $\sigma_2$ are normalized by the quantity $\varepsilon_g V_{max} \delta_g$, which is the amount of charge necessary to cause sticking under the assumptions of: rotor infinitesimally close to the upper stator insulation, $V_1 = 0$, $V_2 = V_{max}$, and $\sigma_1 = \sigma_2$. The nondimensional surface charges are:

$$
\frac{\sigma_1}{\varepsilon_g V_{max}} = \frac{2\delta_g \sigma_1}{\varepsilon_g V_{max}}, \quad \frac{\sigma_2}{\varepsilon_g V_{max}} = \frac{2\delta_g \sigma_2}{\varepsilon_g V_{max}}
$$

(16)

The rotor displacement, $y_r$, is normalized by $\delta_g$. To this point, normalization is straightforward, driven in large part by the idea of making the natural range of the normalized variables be $0$ to $1$. In order to do the same for $v_r$, it would be sensible to normalize $t$ by the amount of time required for the rotor to traverse the gap. This amount of time, however, depends on the amounts of surface charge, and, in any event, cannot be expressed simply. The simplest natural time increment associated with this system is $\rho_0 \delta_g$, the inherent charge relaxation time of the insulator material. This value, therefore, is used to normalize $t$, resulting in:

$$
\bar{y}_r = \frac{y_r}{\delta_g}, \quad \bar{t} = \frac{t}{\rho_0 \varepsilon_i}, \quad \bar{v}_r = \frac{\rho_0 \varepsilon_i v_r}{\delta_g}
$$

(17)

In terms of the normalized variables, the state equations are:

$$
\frac{\bar{\delta}_1}{\bar{t}} = \left[ 1 - \bar{y}_r \right] \frac{\sigma_1}{\bar{V}_1} + \left[ \frac{2}{1 + c - \bar{y}_r} \right] \bar{V}_1
$$

(18)

$$
\frac{\bar{\delta}_2}{\bar{t}} = \left[ 1 + \bar{y}_r \right] \frac{\sigma_2}{\bar{V}_2} + \left[ \frac{2}{1 + c + \bar{y}_r} \right] \bar{V}_2
$$

(19)

$$
\frac{\bar{y}_r}{\bar{t}} = \bar{v}_r, \quad \left| \bar{y}_r \right| \leq 1 - \bar{d}
$$

(20)

$$
\frac{\bar{v}_r}{\bar{t}} = A \left[ \frac{2\bar{V}_1 + c\bar{\sigma}_1}{1 + c - \bar{y}_r} \right]^2 - \left[ \frac{2\bar{V}_2 + c\bar{\sigma}_2}{1 + c + \bar{y}_r} \right]^2
$$

(21)

where the constants $A$, $c$, and $\bar{d}$ are defined as:

$$
A = \frac{\varepsilon_g (\rho_0 \varepsilon_i V_{max})^2}{8 \mu_0 \delta_g^3}
$$

(22)

$$
c = \frac{\mu_0 \delta_g}{\varepsilon_g \varepsilon_i}
$$

(23)

$$
\bar{d} = \frac{d}{\delta_g}
$$

(24)
Thus, by normalization, the number of free parameters is reduced to three. The parameter \( A \) may be understood as the square of the ratio of the characteristic time associated with charge relaxation \( (\rho/e_s) \) to the characteristic time associated with gap traversal \( (2\delta_y/2\mu_e e_s)^{1/2}/V_{\text{max}} \).

**Actuator Control**—The two inputs to the state equations, \( V_1 \) and \( V_2 \), control the rotor motion and forces produced by the model. As described above and shown in Figure 4, the voltages are switched upon rotor/stator contact—this form of motor control is known as commutation. The model assumes ideal knowledge of contact time. In practice, this knowledge would be obtained from a proximity sensor, such as a contact or capacitance based switch. From the point of view of numerical simulation, commutation is simple: when the rotor contacts a stator, that nondimensional voltage is set to ground and the opposite stator voltage is set to unity. However, it should be understood that the absolute switching time depends on the transient solution to the state equations, which changes with each cycle as surface charge builds or decays (and which would also change with loading conditions). Thus, commutation is a form of "asynchronous" control.

![Diagram](image)

**Figure 4:** Actuator control scheme (commutation) used in simulations

**Simulation and Results**—Given the commutation scheme, solutions to the state equations change for each combination of the parameters \( A \), \( c \), and \( \delta \). In practice, each parameter is confined to some range of values. The upper and lower bounds on each design parameter are determined by either a physical (e.g., \( V_{\text{max}}/\delta_e \) limited by breakdown field strength) or technological (e.g., \( i_L \) limited by physical vapor deposition process) limit. Substituting appropriate limits into equations 22-24, the largest conceivable ranges for \( A \), \( c \), and \( \delta \) are:

\[
1 \times 10^{-11} < A < 1 \times 10^{25} \\
1 \times 10^{-4} < c < 1 \times 10^{-1} \\
1 \times 10^{-5} < \delta < 1 \times 10^{-1}
\]

Although such an expansive range for \( A \) allows many design possibilities, it is also unwieldy. Therefore, this range is further limited. For very small values of \( A \), charge build-up is fast enough that the insulator may as well not be present: breakdown is almost certain to occur. For very large values of \( A \), charge accumulation is slow, but essentially permanent: in practice, the effects of spurious sources of charge, such as breakdown, are likely to be felt to a significant degree. Large values of \( A \) are also difficult to deal with in simulation, because many cycles are needed before a steady state behavior is reached. Simulation experience suggests that a good range for \( A \) is:

\[
1 \times 10^{-4} < A < 1 \times 10^{5}
\]

The principal goal of the simulation is to delineate those regions of the \( A, c, \delta \) parameter space in which the rotor "sticks." The rotor sticks to a stator when the force from the charge on the contacting insulator overcomes the force of the charge and voltage produced by the opposite stator.
An analytical expression for sticking can be derived from the state equation for rotor velocity (eq. 21). The sticking condition depends on parameters \( c \) and \( \bar{d} \), as well as the rotor position:

\[
\sigma_2 \geq \frac{2 + c \cdot \bar{d}}{c + \bar{d}} \sigma_1 - \frac{2}{c}, \quad y_r = 1 - \bar{d}
\]

(25)

\[
\sigma_2 \geq \frac{2 + c \cdot \bar{d}}{2 + c \cdot \bar{d}} \sigma_1 + \frac{2}{c}, \quad y_r = \bar{d} - 1
\]

The entire parameter space is explored via simulation to discover the parameter sets for which sticking occurs. The results are summarized by figure 5, which shows lines of constant \( \bar{d} \) within the \( A/c \) space which lie on the borderline between sticking and non-sticking. In effect, figure 5 is a topographical map of the two-dimensional manifold within \( A/c/\bar{d} \) space which separates the regions of sticking and non-sticking. The sticking region is "beneath" this manifold.

The results in Figure 5 are better understood by examining individual parameter sets in greater detail. For the parameters \( A=1, c=0.01 \), and \( \bar{d}=0.01 \) the cycle sticks and for the parameters \( A=1, c=0.006 \), and \( \bar{d}=0.01 \) the cycle remains continuously free, according to Figure 5. The following example analyses, though specific to the afore said parameters, illustrate expected results in all sticking and non-sticking areas of the \( A/c/\bar{d} \) space. Figure 6 clearly shows the effect of sticking on rotor position. For the sticking cycle, the rotor is held momentarily motionless, just after contact with a stator, while charge decays. Although the voltage levels are switched precisely at contact, motion and usable work do not immediately follow.

Although one of the example parameter sets sticks, both have stable cycles. Figure 7 shows the normalized force at the beginning of each cycle for the sticking and non-sticking parameter sets, and for two different sets of initial conditions on charge. The first set of initial conditions is determined from (25), setting \( \sigma_1=\sigma_2 \) (with \( y_r=\bar{d}-1 \)), so that sticking is initially guaranteed. For both parameter sets, the force at the beginning of each cycle quickly reaches a constant value, indicating that a "steady state" condition has been reached. The sign of the force indicates that one parameter set exhibits sticking in steady state, while the other does not. The second set of initial conditions is chosen such that sticking is initially guaranteed not to occur. Again, the force levels quickly settle to the same constant values. Since the state equations exhibit the same steady-state solutions (at a

![The "Sticking" Condition in c/A Parameter Space with Variation by Parameter \( \bar{d} \)](image)

Figure 5: Lines of constant \( \bar{d} \) mark the boundary for sticking cycles in the \( c/A \) parameter space.

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Figure 6: Transient rotor position for a sticking and stick free cycle. For particular point in the cycle, for varying initial conditions, the associated limit cycles are termed stable. Note, different parameter sets may take more cycles to reach steady state operating conditions.

Figure 7: Stability of cycles with and without sticking.
The stability of the attachment/detachment cycles can also be shown in the pseudo charge "phase plane"\textsuperscript{1}. Although rotor position and velocity are not included, these variables are always bounded; indeed, their steady-state oscillation induces a steady state oscillation in the values of the surface charge. Figures 8 and 9 show the charge "phase plane" for the two example sets. In both cases, limit cycles symmetric about the $c_1=c_2$ axis quickly converge. In the first case, the charging cycle is completely inside the sticking condition (25) boundaries. The charging cycle of the second parameter set clearly crosses the associated sticking condition, indicating an excess of charge on the insulator surfaces. Since even the sticking cycle is stable, perhaps a specified amount of sticking could be designed advantageously into the micro actuator. In general, however, the effects of sticking are negative: motor speed and force production are reduced, and the chances of breakdown are increased. Further, the model has simulated only no load conditions, and slower operation due to a load may exacerbate the delay due to sticking.

![Charge "Phase Plane" of Cycle with No Sticking](image)

Figure 8: Charge "Phase Plane" of a parameter set with no sticking.

\textsuperscript{1}A true phase plane for this system would be four dimensional (the dimension of the state space).
5. Discussion and Conclusions

**Implied Design Tradeoffs**—The most important conclusion to be drawn from the previous section is that charge buildup due to RC mechanisms cannot be neglected in the design process. Moreover, the simulation results provide important criteria for the design of linear micro actuators. Making up the three mathematical parameters that "map" when RC charging will become critical are no less than six design variables that can be manipulated to define the actuator. This redundancy seems to offer flexibility; however, several implicit design tradeoffs must be understood. Referring to Figure 5, c should be small in order to increase the useful range of $d$ and $A$ parameters. To make $c$ low, the insulator thickness can be reduced until electric field breakdown in the insulator occurs. Alternatively, the gap can be increased, but this also reduces the value of $A$ by a cubed power. An undesirable consequence is that the force and velocity production of the actuator drop rapidly. The maximum voltage can be increased to compensate $A$ for larger gap sizes, but the field increase will quickly overtake the maximum tolerable field before breakdown in the gap occurs. The insulator resistivity can be increased (to maintain a larger $A$), but this will not improve force and speed production. Even when the breakdown field strengths are avoided, the RC charging phenomenon ensures that designing a stick free motor for useful work output will be challenging.

**Example Actuator Design**—To understand better the simulation results and implications, an example actuator is here described. Physical vapor deposition is used to place a 1 μm layer of ceramic insulation ($p=10^{11}$) on a metallic conductor finished with 0.5 μm diamond polishing compound ($d=0.5μm$). For a gap size of $δ_g=10μm$ and an insulator relative permittivity of 2, the normalized contact separation is $d/δ=0.025:1$ and $c=0.1$, which leaves a range on the final parameter, $10<d<1000$. The rotor inertia per area for a thin layer of copper deposited on a lighter polymer substrate is $\mu=0.05\text{ kg/m}^2$. Breakdown limits the maximum voltage to $V_{max}=100\text{V}$, which forces $A=8.7\times10^8$, definitely out of range. If the resistivity could be adjusted to $\rho_r=10^8$, then $A=10^3$. For this set of parameters, the material time constant is $\tau=0.001\text{s}$ and the charging cycle never crosses the sticking boundary.
**Future Work**—Several refinements in the model may better elucidate the design criteria. If the actuator operates in an air-filled environment, the effects of squeeze film damping must be included in the model. Currently the only source of charge is from RC effects; contact electrification and breakdown should also be modeled and experimentally investigated. Real motors generally operate with a more complicated commutation scheme. For instance, a commutation scheme for the actuator in Figure 2 would include a wait period after the rotor contacts a stator (this wait enables the remaining length of the rotor to cross the gap) before the voltage switches. The length of the wait period, as well as the applied voltage during the wait period, become additional design variables.

Currently, experiments are being designed to verify the results and explore the implications of the model. The emphasis is on the mechanisms of charge accumulation, with effort directed at isolating RC, breakdown, and contact electrification effects. The apparatus, controller, and data collection protocol will be flexible enough to investigate future refinements to the model. The synthesis of modeling and experimental experience should further elucidate criteria that may be used to guide the design of electrostatic micro actuators.

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**References**


