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POWER AND IMPEDANCE SCALING IN BILATERAL MANIPULATION

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Abstract

A power and impedance scaling bilateral manipulator (such as an "extender" or a "macro-micro bilateral manipulator") can greatly enhance the manual capabilities of a human operator, but it can also compromise the inherent stability of the operator. In this paper, a condition for the robust stability of an operator/bilateral manipulator/environment system is derived using the structured singular value (μ). The application of this condition is illustrated with several examples of power and impedance scaling via a two-channel bilateral manipulator.

1. Introduction

An important class of bilateral telemanipulators is that in which the master and slave operate on very different length, force, and power scales. This class includes strength-increasing "man-amplifiers" or "extenders" as well as dexterity-increasing "macro-micro bilateral manipulators" (MMBMs). Because they are intrinsically power-scaling, such devices may be roughly described as generalized mechanical amplifiers. A fundamental difference, however, between an operational amplifier (for instance) and a power-scaling bilateral manipulator is that the former is an information-processing (unilateral) device, and as such, features high input impedance and low output impedance, while the latter is a power-processing (bilateral) device, and as such, features input and output impedances that are roughly matched to those of the operator and environment, respectively. Matched impedances imply *significant* energetic interaction, and it is coping with this energetic interaction that presents one of the greatest challenges of bilateral manipulation — with or without power amplification [3,11,12,17].

The principal contribution of this paper is a condition for the robust stability of a power-scaling bilateral manipulator coupled to an environment that is passive, but otherwise arbitrary.

Power Scaling — The traditional concept of a telemanipulator is that of a machine which, by enabling remote manipulation, ensures the safety of a human operator. A distinct, though not mutually exclusive, concept is that of a machine which enhances a human operator's ability to manipulate. This idea appears to date to the early 1960s and the "man amplifiers" built at General Electric [15]. Man amplifiers were intended to give the ordinary man extraordinary strength, presumably supplanting the need for an array of powered tools, such as the hydraulic floor jack. It is interesting to note that the same period gave rise to cybernetic prostheses, which were to give amputees benefits beyond those of body-powered prostheses, by providing greater strength and more natural means of control [19]. Neither man amplifiers nor cybernetic prostheses met with much initial success, judged in terms of user acceptance. Common to both seemed to be the problem of mental fatigue: they simply required too much concentration on the part of the operator.

In the 1980s, both saw something of a rebirth. Kazerooni introduced the "extender," which is similar to the man amplifier; however, by being intimately connected to the powered limb of the extender, the operator communicates with it via *both* power and information [12]. In effect, the extender is a power-assist, not unlike a power steering system. Childress and coworkers demonstrated that a very similar concept, called "extended physiological proprioception" (EPP), provided significant benefits for the control of upper-extremity prostheses [8]. EPP requires that a 1:1 relationship be maintained between the *position* of a control site on the operator and the *position* of the artificial limb. For instance, the forearm of an elbow prosthesis may be connected to a shoulder harness via a Bowden cable, "tying together" certain movements of the shoulder and rotation of the elbow joint, even though the elbow receives a power assist in proportion to the force on the cable. In this way, both force and position are experienced with the operator's own (shoulder) proprioceptors, establishing a very intimate man-machine relationship. It is the goal of neither extenders nor EPP to provide a limb replacement, but rather it is the goal of both to provide a very natural limb *extension*, in much the same sense that a tennis racket is an extension to the arm of an accomplished player.

Impedance Scaling — It is the rigidity of a tennis racket that preserves the relationship between the location of its strings and the location of the player's hand; however, it is the compliance of the racket that is equally important in providing the player "feel" and control over his or her shots. Recently, Raju introduced the concept of tuning a bilateral manipulator's impedance — at both master and slave ports — to optimize performance, much as a tennis racket is sized and strung to best suit the player's taste and, presumably, performance [17]. This concept should prove to be particularly important in the context of power scaling telemanipulation, where the operator and environment may have very different impedances, and some care must go into matching them via the bilateral manipulator.

Macro-Micro Bilateral Manipulation — The author's primary interest is in bilateral micromanipulation, which may be used to provide dexterity enhancement in microsurgery or microelectronics assembly. The development of a manual interface for a macro-micro bilateral manipulator (MMBM) is described in [14].

The approach that has been taken to bilateral micromanipulation is that an operator be able to use the MMBM as he or she would any other *hand tool.* Although the MMBM will consist of separate master and slave manipulators, operating on very different length and power scales, the operator should have the perception that it is a single entity, identifiable as a knife or as tweezers, for instance. This philosophy is somewhat like EPP, in that a tool acts as an extension to a human's limb. The questions arise: what motion and force relations between the master and slave manipulators will optimize the operator's perception of tool use (and, presumably, the operator's performance); and, what constraints on MMBM behavior are required to ensure stability/robustness? The main contribution of this paper is an answer to the latter question; however, the approach that has been taken to the former question will be briefly reviewed.

The approach to optimizing the perception of tool use is the following: the slave/environment system should be made to appear as a geometrically similar macro system of appropriate dynamic behavior. Thus, the slave and microenvironment should be viewed via a display system that scales up their image by a fixed length ratio ($1/k_\phi$) that relates macro to micro dimensions. Moreover, movements of the slave should be related to movements of the master by the length scale k_ϕ in order to maintain the perception of a 1:1 relationship between the position of the operator's hand and the position of the tool.

The force relation, however, is more complicated. Suppose that the slave/environment system is primarily inertial. Because mass scales as length cubed, it would make sense to amplify the forces measured at the slave by a factor of $k_\varepsilon = k_\phi^{-4}$, so that the impedance felt at the master would be scaled up by a factor of $k_\varepsilon k_\phi = k_\phi^{-3}$ (see Figure 1). In this way, the slave/environment system would also *feel* like a macro system. But suppose now that the slave/environment system also exhibits viscous damping. Viscous forces vary as the area, thus the appropriate force scale would be $k_\varepsilon = k_\phi^{-3}$. However, as both dynamic effects are present in the slave/environment system, which scale should be chosen? Because geometrically similar macro and micro systems are not dynamically similar, no fixed scale factor is likely to optimize the

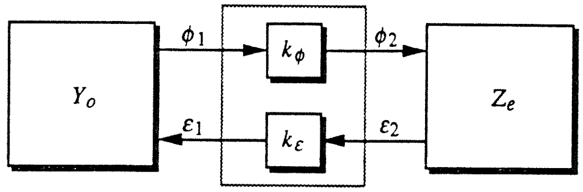

Figure 1. Block diagram of a one degree of freedom teleoperator system with an "ideal" (dynamics-less) power scaling bilateral manipulator. ϕ is a flow variable and ε is an effort; Y_o is the admittance of the operator, and Z_e is the (linear) impedance of the environment. k_ϕ and k_ε are dimensionless, static scaling factors. The "apparent impedance" of the environment is given by $\varepsilon_1(s)/\phi_1(s) = k_\varepsilon k_\phi Z_e(s)$.

perception of tool use. A better approach may be to dynamically "reshape" the slave/environment impedance (e.g., scale up the inertia to a greater extent than the damping) to create a dynamic behavior that is "appropriate" to the geometrically similar macro system. One approach to "impedance shaping" is described in [5]. Although impedance shaping is a potentially powerful approach to optimizing dexterity, it has obvious pitfall in terms of compromising stability/robustness. Thus, in order to design useful impedance shaping bilateral controllers, a powerful robustness criterion is needed. Such a criterion is presented in the next section.

3. Robustness

Problem Statement— It is a matter of common observation that human operators have no difficulty maintaining stability when interacting with all manner of *passive* tools and environments. Actually, humans are even more skilled: some tools, such as rotary sanders and floor waxers, exhibit what are obviously not passive impedances, but humans can, after some training, operate them successfully. Nonetheless, the broadest set of tool/environment behaviors that can, at this point, be characterized as stable under human control, is the set of passive behaviors. Therefore, the following intuitive condition for stability/robustness will be used: *a bilateral manipulator is said to be robust if, when coupled to any passive environment, it presents to the operator an impedance which is passive.* Thus, while the apparent impedance felt through the bilateral manipulator may have a radically different magnitude, or even shape in the frequency domain, than the environment impedance, it must at least be passive.

The assumption will now be made that the environment is a passive, linear time-invariant n-port, but is otherwise completely arbitrary. It will be the subject of future work to treat nonlinear environments and other, perhaps more narrowly defined classes of environments. The mathematical statement of the problem will require that

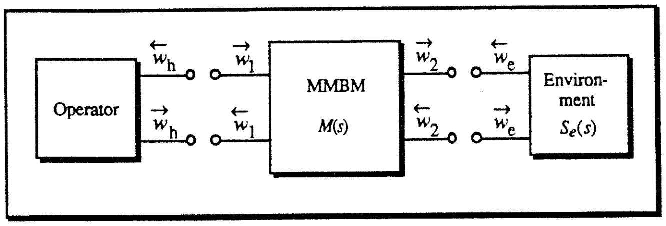

Figure 2. Block diagram showing wave-scattering description of operator, MMBM, and environment. Note that arrow directions indicate *in* and *out*, not left and right.

the bilateral manipulator and environment be described in terms of **scattering matrices**. In a standard impedance, admittance, or hybrid description of a physical system, root power variables — efforts and flows — serve as inputs and outputs; in a scattering description, inwave ($\vec{w}$) and outwave ($\overleftarrow{w}$) variables serve as inputs and outputs. The inwave and outwave are taken to be proportional to the square root of incident power and reflected power, respectively. The relationships between root power and scattering variables, and between hybrid matrices and scattering matrices, are described in a number of standard texts, such as [1], as well as in the tutorial and review by Paynter and Busch-Vishniac [16], and the article by Anderson and Spong [3], which exploits scattering variables to provide a solution to the time delay problem in teleoperation. The value of a scattering description in the present context is that passivity relates to the familiar infinity norm of the scattering matrix:

Passivity — A linear time-invariant n-port with a scattering matrix $S(s)$ (i.e., $\overleftarrow{w} = S(s)\vec{w}$) is passive iff:

1. $S(s)$ contains no poles in the closed right half plane.
2. $\|S(j\omega)\|_\infty \leq 1$.

A proof may be found in [1].

Suppose that the bilateral manipulator and environment are described by the following scattering matrices (see also Figure 2):

bilateral manipulator: $M(s) = \begin{bmatrix} M_{11}(s) & M_{12}(s) \\ M_{21}(s) & M_{22}(s) \end{bmatrix}$;

environment: $S_e(s)$,

where $M(s)$ is partitioned according to master ports (subscript 1) and slave ports (subscript 2). It is then straightforward to show that the sought after conditions are that $M(s)$ contain no poles in the closed right half plane, and that:

$$\|M_{11} + M_{12}S_e(I - M_{22}S_e)^{-1}M_{21}\|_\infty \leq 1 \quad \text{for all } \|S_e\|_\infty < 1$$

The dependence upon the Laplace variable, s, is understood. This condition, however, is not very useful, because the norm must be computed for all $n{\times}n$ S_e of infinity norm less than one. It would be preferable to have a condition in terms of $M(s)$ alone.

The key to finding such a condition lies in recognizing that the conditions sought are equivalent to the conditions for the *stability* of the telemanipulator/ environment when coupled to an operator which is passive, but otherwise arbitrary. This can be understood by replacing the operator block in Figure 2 with an arbitrary passive n-port, then asking the question, under what conditions on $M(s)$ is stability guaranteed? The conditions are the same as those sought here, because a necessary and sufficient condition to guarantee the stability (i.e., all poles lie in the closed left half plane) of a system coupled to an arbitrary strictly passive n-port, is that the system itself appear to be passive [4]. Note that treating the operator as passive is simply a useful artifice, and is not indicative of any assumptions having been made about the operator dynamics.

The problem may now be restated as: what conditions must $M(s)$ satisfy if it is to remain stable (but not exponentially stable) when coupled to a $2n$-port that meets the following criteria:

$$S(s) = \begin{bmatrix} S_o^{n\times n} & 0 \\ 0 & S_e^{n\times n} \end{bmatrix}; \quad \|S\|_\infty \leq 1,$$

but is otherwise arbitrary? Notice that $\|S\|_\infty \leq 1$ if and only if $\|S_o\|_\infty \leq 1$ and $\|S_e\|_\infty \leq 1$. If the coupled system composed of $M(s)$ and $S(s)$ is stable, it follows that the apparent impedance felt at the master ports is passive.

If, for some reason, the operator's admittance is known to be strictly passive according to the definition given in [6], then the exponential stability of the operator states can be guaranteed by the sought-after conditions on $M(s)$.

Problem Solution — This problem is now virtually analogous to the **structured singular value** problem considered by Doyle [9] and others [7,10]. If $X_\infty(s)$ is defined as the class of $2n$-port block diagonal matrices (same structure as S, above) with no restriction on infinity norm, then the structured singular value of M, $\mu(M)$, is defined as follows:

$$\mu(M) = \begin{cases} 0 & \text{if no } \Delta \in X_\infty \text{ solves } \det(I + M\Delta) = 0 \\ \left(\min_{\Delta \in X_\infty} \left\{\bar{\sigma}(\Delta) \mid \det(I + M\Delta) = 0\right\}\right)^{-1} & \text{otherwise} \end{cases}$$

where $\bar{\sigma}(\Delta)$ is the maximum singular value of Δ. Necessary and sufficient conditions for stability can now be stated in terms of $\mu(M)$:

Coupled Stability for Block Diagonal 2n-Port Environments — The $2n$-port system with scattering matrix $M(s)$ will be guaranteed to remain stable when coupled to an arbitrary, passive, block diagonal $2n$-port ($S(s)$, above) iff:

$$\sup_{\omega} \mu(M(j\omega)) \leq 1$$

With the assistance of the multivariable Nyquist Theorem [13], the proof of this theorem is straightforward, as the definition of $\mu(M)$ is essentially tautological. It may appear, in fact, that the introduction of $\mu(M)$ has been counterproductive, as all "perturbations" $\Delta \in X_\infty$ must be considered. Certain properties of $\mu(M)$, however, make it possible to arrive at a condition that may be computed in terms of $M(s)$ alone.

One rather obvious property of $\mu(M)$ is the following:

$$\mu(M) \leq \bar{\sigma}(M). \tag{1}$$

Consider now a diagonal matrix, D, with the following structure:

$$D = \begin{bmatrix} d_1 I^{n \times n} & 0 \\ 0 & d_2 I^{n \times n} \end{bmatrix},$$

where $d_1 > 0$ and $d_2 > 0$. It is easy to see that $D^{-1}\Delta D = \Delta$; therefore, for the purposes of computing μ, the perturbation Δ and the perturbation $D^{-1}\Delta D$ are equivalent. Moreover, as is easily demonstrated via block diagram manipulation, it is also equivalent to treat DMD^{-1} as the plant and Δ as the perturbation. Thus, $\mu(DMD^{-1}) = \mu(M)$. It follows that:

$$\mu(M) \leq \bar{\sigma}(DMD^{-1}). \tag{2}$$

Doyle has proved that, when $\bar{\sigma}(DMD^{-1})$ is minimized over all allowable D, the equality in equation 2 holds; i.e.:

$$\mu(M) = \inf_{D} \bar{\sigma}(DMD^{-1}). \tag{3}$$

Moreover, the infimum is convex. Using this result, the robustness criterion may be written:

Robustness for 2n-Port Teleoperators — A $2n$-port teleoperator with scattering matrix $M(s)$ will be guaranteed to exhibit a passive impedance at the master ports when coupled to a passive, but otherwise arbitrary environment at the slave ports, iff:

$$\sup_{\omega}\left(\inf_{\alpha > 0} \bar{\sigma}\left(\begin{bmatrix} M_{11} & \alpha M_{12} \\ \frac{1}{\alpha} M_{21} & M_{22} \end{bmatrix} \right)\right) \leq 1.$$

With this criterion, coupled stability can be assessed in terms of $M(s)$ and a single scalar parameter, α. Although α cannot, in general, be determined analytically, a variety of efficient numerical methods have been suggested [9,10].

4. Examples

Multichannel Amplification — An important subclass of $M(s)$ is that of $2n$-port amplifiers, as illustrated in Figure 3. This subclass was previously considered by Anderson [2]. For such an amplifier, the scattering matrix can be shown to be:

$$M = \begin{bmatrix} \mathrm{diag}\left\{ \frac{K_\phi^i K_\varepsilon^i - 1}{K_\phi^i K_\varepsilon^i + 1} \right\} & \mathrm{diag}\left\{ \frac{2K_\varepsilon^i}{K_\phi^i K_\varepsilon^i + 1} \right\} \\ \mathrm{diag}\left\{ \frac{2K_\phi^i}{K_\phi^i K_\varepsilon^i + 1} \right\} & -\mathrm{diag}\left\{ \frac{K_\phi^i K_\varepsilon^i - 1}{K_\phi^i K_\varepsilon^i + 1} \right\} \end{bmatrix}, \tag{4}$$

where each submatrix is $n \times n$. If, for each channel, the power scale factor, $K_\phi^i / K_\varepsilon^i$, is equal to β, then by selecting $\alpha = \sqrt{\beta}$, it can be shown that:

$$M_\alpha = \begin{bmatrix} M_{11} & \alpha M_{12} \\ \frac{1}{\alpha} M_{21} & M_{22} \end{bmatrix} = \begin{bmatrix} \mathrm{diag}\left\{ \frac{K_\phi^i K_\varepsilon^i - 1}{K_\phi^i K_\varepsilon^i + 1} \right\} & \mathrm{diag}\left\{ \frac{2\sqrt{K_\phi^i K_\varepsilon^i}}{K_\phi^i K_\varepsilon^i + 1} \right\} \\ \mathrm{diag}\left\{ \frac{2\sqrt{K_\phi^i K_\varepsilon^i}}{K_\phi^i K_\varepsilon^i + 1} \right\} & -\mathrm{diag}\left\{ \frac{K_\phi^i K_\varepsilon^i - 1}{K_\phi^i K_\varepsilon^i + 1} \right\} \end{bmatrix}, \tag{5}$$

and that $\sqrt{\beta}$ is the value of α that minimizes the maximum singular value of M_α. M_α can be recognized as the scattering matrix of a multiport transformer, which is a unitary matrix [1]. The structured singular value of a unitary matrix is one, and the multi-channel amplifier meets the coupled stability criterion. However, if $K_\phi^i / K_\varepsilon^i \neq K_\phi^j / K_\varepsilon^j$ for any i,j, indicating that the power

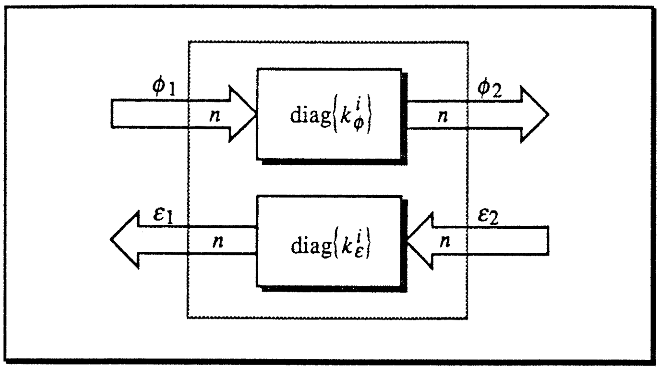

Figure 3. $2n$-port amplifier, $i = 1 \ldots n$.

scalings on any two channels are unequal, then the coupled stability criterion will not be met. In the next section, an example is given of a bilateral manipulator that admits different power scalings on different channels.

Two-Channel MMBM — Consider the 4-port (2 channel) MMBM model shown in Figure 4. Each channel consists of a master (M_m), an impedance-controlled slave (M_s, K_s, B_s), and a bilateral controller consisting of constant velocity and force scale factors (K_ϕ, K_ε). Despite the simplicity of the controller and the absence of inter-channel coupling, this model is sufficiently rich to illustrate several of the consequences of manipulator dynamics and scale factor selection on stability/robustness in a multi-channel setting.

Case 1 — The slave is critically damped on both channels, the power scale factors are the same on both channels

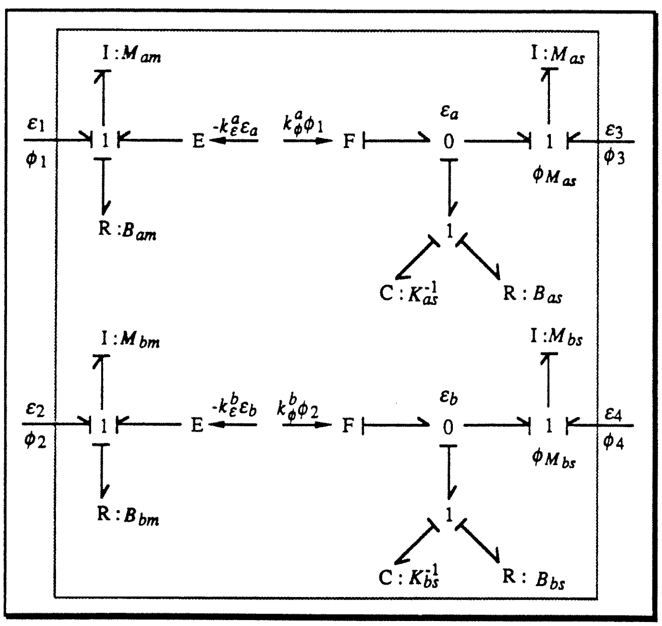

Figure 4. Bond graph model of a two-axis MMBM (enclosed by box). Two ports (1 and 2) connect it to the operator and two ports (3 and 4) connect it to the environment.

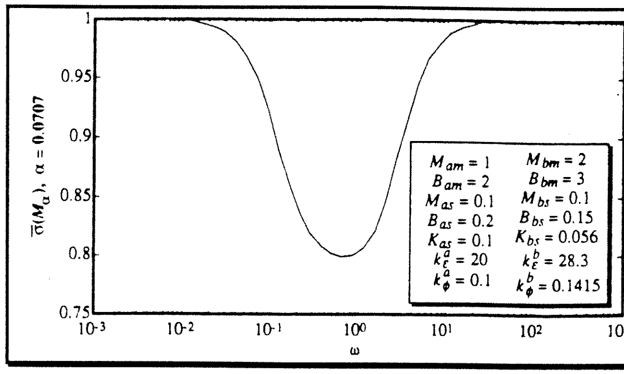

Figure 5. Magnitude of maximum singular value of M_α (defined in equation 5) versus frequency. Note that $\|M_\alpha\|_\infty \le 1$ is a sufficient condition for $\sup_\omega (\mu(M)) \le 1$.

($K^a_\phi/K^a_\varepsilon = K^b_\phi/K^b_\varepsilon = 200$), but the impedance scale factors are different ($K^a_\phi K^a_\varepsilon = 2$, $K^b_\phi K^b_\varepsilon = 4$). The MMBM satisfies the coupled stability criterion, as expected (Figure 5).

Case 2 — This case (Figure 6) is similar to the last, except that the power scaling on channel B is twice that on channel A. The MMBM does not satisfy the coupled stability criterion. It is important to note that the violation occurs at low frequencies only — at higher frequencies sufficient energy is dissipated in the slave damping, and at the highest frequencies, the behavior of the MMBM is dominated by the master and slave inertias. Further investigation has shown that the violation at low frequency cannot be avoided by changing any of the damping factors, or by adding damping effects that couple the two channels. Indeed, for this MMBM, violation is unavoidable if the power scales on the two channels are not the same.

To understand why this must be, consider the MMBM that would be obtained by replacing the 4-port amplifier of this example by two rigid links, one connecting master and slave on channel A, and one connecting master and slave on channel B. Such an MMBM would be passive, and would satisfy the robustness criterion. However, at low enough frequency, each of the channels would appear to be essentially a rigid link connected to ground by the dampers B_{am} and B_{bm}. By attaching to each side a 2-port lossless system of sufficiently low resonant frequency, and by carefully tuning the impedances of these 2-ports so that they could react against one another across the MMBM while producing no net force on the MMBM, the entire system could be made to resonate almost indefinitely. Now if the two links were replaced by amplifiers of different power scale factors, it would be possible for energy to be amplified without bound by passing from channel to channel via the lossless 2-ports. Although this is a failing of any bilateral manipulator that acts as a rigid link at low enough frequency, a more

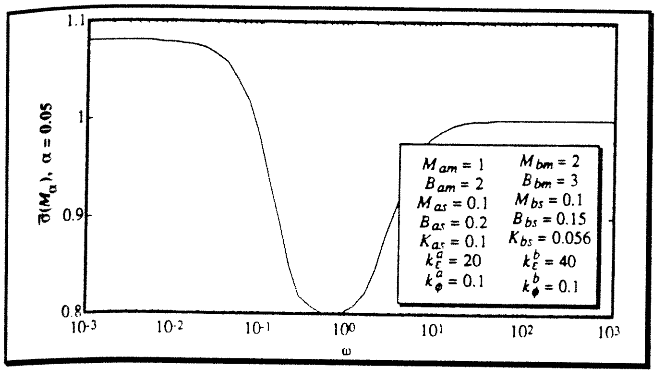

Figure 6. Case 2. $\alpha = 0.05$ corresponds approximately to that which minimizes $\bar{\sigma}(M_\alpha)$ at low frequency.

serious failing is a classification of potential environments that permits such pathological cases.

Case 3 — This case is similar to the previous one in that the power scaling factors are different on the two channels; however, the stiffness of the slave on the two channels has been set to zero (the slave is velocity controlled). As Figure 7 indicates, the robustness criterion is satisfied, even though the power scales are different by a factor of nearly 42. This can happen because energy cannot flow through the MMBM unattenuated at any frequency. On the other hand, this MMBM does not maintain a fixed position relationship between master and slave at low frequency.

5. Conclusions

A necessary and sufficient condition for the apparent passivity of a $2n$-port bilateral manipulator coupled to a linear time-invariant passive, but otherwise arbitrary, environment was given. Several examples were given of the application of this condition. The examples indicated that a practical MMBM would require nearly identical power scalings on each channel, though the impedance scalings might be radically different.

Future research will address the extension of this result to include nonlinear environments; improved classifications of environment behavior that will encode additional information, such as bounds on impedance magnitude; and the development of robust impedance shaping bilateral controllers.

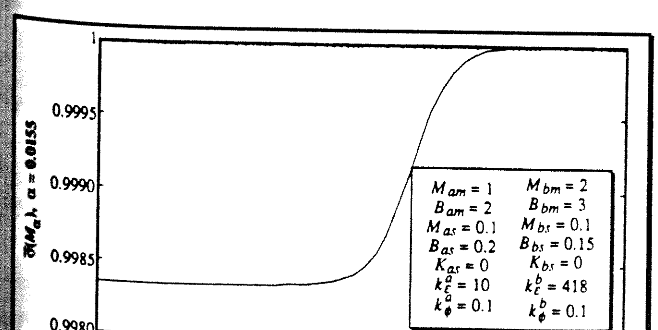

Figure 7. Case 3.

Acknowledgement

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