Stability of manipulators interacting with unstructured and structured dynamic environments

J. Edward Colgate

Department of Mechanical Engineering, Northwestern University, Evanston, IL 60208-3111, U.S.A.

Abstract

Often it is desirable to guarantee that a robot or teleoperator will remain stable when coupled to any member of a particular class of environments. Input/output techniques, including the Small Gain Theorem and the Small μ Theorem, provide powerful tools for establishing such guarantees. In this paper, classes of environments that can be treated with input/output techniques resulting in necessary and sufficient stability conditions are identified.

Special attention is paid to two physically meaningful classes: passive environments and structured passive environments. Associated with these, necessary and sufficient conditions for the coupled stability of robots and teleoperators, respectively, are presented. The utility of these results is illustrated with two examples. In the case of robotics, robustness measures for force feedback compliance control are discussed. In the case of bilateral manipulation, the problem of robust power amplification is addressed.

1. INTRODUCTION

Robots and teleoperators are fundamentally intended to serve as machines for manipulation of the physical world. In the widest sense, "manipulation" refers to physical interaction, characterized by contact with broad classes of often poorly characterized environments. If a manipulator and the environment which it contacts are collectively considered a physical system, then this is surely a system of highly uncertain dynamic behavior. Yet the job of the robot or teleoperator designer may be viewed as that of ensuring stability as well as a specified level of performance, despite this uncertainty.

In this paper, the problem of ensuring manipulator/environment stability ("coupled stability") will be addressed. The problem of ensuring performance is more difficult, in large part because performance specifications are likely to change with the environment. Coupled stability will be addressed with the use of input-output techniques, including the Small Gain Theorem [6] and Small μ Theorem [7]. The former applies to certain classes of nonlinear as well as linear systems, and a nonlinear version of the latter has been developed [2]. Nonetheless, to simplify exposition, and to permit some results that would not be possible otherwise, only linear analysis will be used.

2. COUPLING AS FEEDBACK

When a manipulator couples to a physical environment, a bilateral interaction is created that can be modeled as a feedback loop. Bilateral interaction is most often described in terms of dynamic operators called impedances and admittances which map flows (ϕ) to efforts (ε) and efforts to flows, respectively. "Flow" refers to a generalized vector of velocities and angular velocities, and "effort" refers to a generalized vector of forces and torques. The inner product of effort and flow is the instantaneous power input to the associated physical system. In terms of a manipulator's impedance, $Y_m(s)$, and an environment's impedance, $Z_e(s)$, coupling creates the feedback structure shown in Figure 1. In manipulation, it is generally known only that the environment impedance belongs to a certain class.

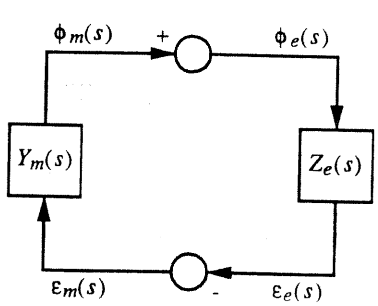


Figure 1.

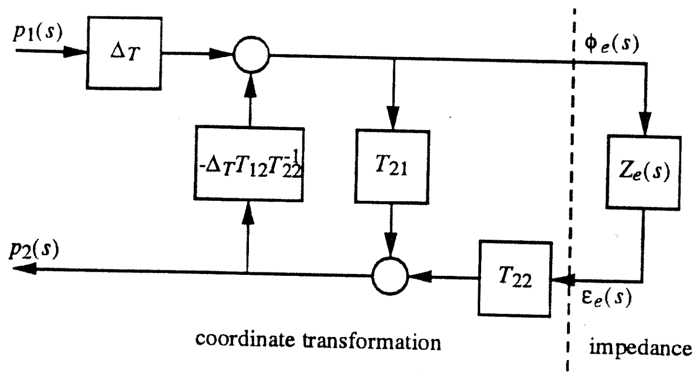


Figure 2.

3. COUPLED STABILITY VIA THE SMALL GAIN THEOREM

The Small Gain Theorem guarantees the stability of the feedback system in Figure 1 if $\|Z_e(s)Y_m(s)\|_\infty < 1$, where $Z_e(s)$ and $Y_m(s)$ are assumed stable (the isolated stability of the manipulator and environment is a key assumption of the theory presented here). Because the infinity norm is submultiplicative, stability is guaranteed if:

$$\|Z_e(s)\|_\infty \leq \gamma \quad \text{and} \quad \|Y_m(s)\|_\infty < 1/\gamma \tag{1}$$

or, more generally, if:

$$\|W^{-1}Z_e\|_\infty \leq 1 \quad \text{and} \quad \|Y_m W\|_\infty < 1 \tag{2}$$

where the argument s has been dropped for brevity, and where $W(s)$ is a scalar transfer function.

As an example, suppose that the class of environments consists of springs (springs are considered stable in the present context) having stiffness k less than some maximum value, k_{max}. Then $Z_e(s) = k/s$, $0 \leq k \leq k_{max}$. A good choice for the weighting function would be $W(s) = k_{max}/s$. It is then clear that $\|W^{-1}Z_e\|_\infty = \|k/k_{max}\|_\infty \leq 1$, and that coupled stability can be guaranteed by $\|Y_m k_{max}/s\|_\infty < 1$. This result, however, is rather conservative because the class of environments for which $\|W^{-1}Z_e\|_\infty \leq 1$ is much broader than the actual class of springs. For instance, a spring of stiffness $-k_{max}$ is not in the actual environment set, yet is a member of the environment set used for establishing coupled stability, and will impose severe limits to the class of acceptable manipulator admittances.

This example illustrates a very important point — the Small Gain Theorem takes no phase information into account, and therefore assumes complete phase uncertainty in the environment class. But admittances and impedances are generally not completely phase uncertain. For instance, the phase of a passive environment's impedance must be greater than -90° and less than +90°. Often, however, there is a way around this difficulty. Impedances and admittances arise when efforts and flows are used to describe bilateral interaction — yet, there are many other legitimate coordinate selections.

4. LINEAR FRACTIONAL TRANSFORMATIONS

Consider the change of coordinates from $[\phi_e\ \varepsilon_e]^T$ to $[p_1\ p_2]^T$ defined by the nonsingular linear transformations:

$$\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} \phi_e \\ \varepsilon_e \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} \phi_e \\ \varepsilon_e \end{bmatrix} = \begin{bmatrix} \Delta_T & -\Delta_T T_{12} T_{22}^{-1} \\ -T_{22}^{-1} T_{21} \Delta_T & T_{22}^{-1} + T_{22}^{-1} T_{21} \Delta_T T_{12} T_{22}^{-1} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} \tag{3}$$

where $\Delta_T = (T_{11} - T_{12}T_{22}^{-1}T_{21})^{-1}$. If $\phi_e(s)$ and $\varepsilon_e(s)$ are related by an impedance operator $Z_e(s)$, then $p_1(s)$ and $p_2(s)$ are related as follows (see Figure 2 for the associated block diagram):

$$p_2(s) = (T_{21} + T_{22}Z_e(s))(T_{11} + T_{12}Z_e(s))^{-1}p_1(s) = A_e(s)p_1(s) \tag{4}$$

The operator $A_e(s)$ is a linear fractional transformation (LFT) of $Z_e(s)$. It is, in fact, a type of conformal mapping that takes circular regions of the Z_e-plane to circular regions of the A_e-plane, where a half-plane is considered a circle of infinite extent.

As indicated above, the impedances of passive systems are restricted in phase to $\geq$-90° and $\leq$+90°; moreover, they are not restricted in amplitude. Therefore, the Nyquist plot of a passive system's impedance must lie in the closed right half plane: $\mathrm{Re}\{Z_e(j\omega)\} \geq 0$. While this condition is not completely phase uncertain, an LFT can transform it to one that is. The LFT and associated operator are:

$$\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} I & I \\ -I & I \end{bmatrix} \begin{bmatrix} \phi_e \\ \varepsilon_e \end{bmatrix}; \quad S_e(s) = (Z_e(s) - I)(Z_e(s) + I)^{-1} \tag{5}$$

This LFT is known as the Mobius or bilinear transformation, and maps the right half plane to $\mathcal{D}$, the unit disk centered at the origin. The p's are known as *scattering variables*, and $S_e(s)$ is known as a scattering operator. Thus, in terms of scattering operators, a passive system is bounded in magnitude and completely uncertain in phase.

The question arises, how is the manipulator description to be modified so that the coupled stability properties of the original feedback loop are unchanged? It can be shown that the interaction coordinates should be transformed as follows:

$$\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} T_{11} & -T_{12} \\ -T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} \phi_m \\ \varepsilon_m \end{bmatrix} \tag{6}$$

In the case of scattering variables:

$$\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} I & -I \\ I & I \end{bmatrix} \begin{bmatrix} \phi_m \\ \varepsilon_m \end{bmatrix} \quad \text{and} \quad S_m(s) = (Y_m(s) - I)(Y_m(s) + I)^{-1} \tag{7}$$

Thus, the manipulator's admittance is transformed into a scattering matrix. The immediate conclusion is that a sufficient condition for coupled stability with passive environments is: $\|S_m\|_\infty < 1$; in other words, that the manipulator behave passively. While this may seem to be a conservative condition, it is many cases far less so than conditions based on impedance/admittance formulations.

Despite the utility of the LFT and Small Gain Theorem, they deal with disk-type environment descriptions only. It is not always possible to describe a class of environments as a disk, even using an LFT, nor is it desirable to have the Small Gain Theorem as the sole foundation for a theory of coupled stability.

5. FURTHER EXTENSIONS

Consider the class of inertialess, passive mechanical systems. Nyquist plots of the impedance of such systems must lie in the fourth quadrant; in other words, phase uncertainty is limited to $-90° \leq \angle Z_e(j\omega) \leq 0°$. Such an area cannot be mapped by any LFT to $\mathcal{D}$, only to a semicircle within $\mathcal{D}$. However, the fourth quadrant can be considered the intersection (logical "and") of two half planes ($\mathrm{Re}\{Z_e(j\omega)\}\geq 0$ and $\mathrm{Im}\{Z_e(j\omega)\}\leq 0$, termed $\mathcal{P}_1$ and $\mathcal{P}_2$, respectively), each of which can be mapped to $\mathcal{D}$ by an appropriate LFT. Corresponding to each of these mappings ($\mathcal{P}_1 \rightarrow \mathcal{D}$ and $\mathcal{P}_2 \rightarrow \mathcal{D}$) will be a characteristic equation preserving mapping of Y_m: $Y_m \rightarrow S_{m1}$ and $Y_m \rightarrow S_{m2}$. A necessary and sufficient condition for coupled stability is: either $S_{m1} \in \mathcal{D}$ or $S_{m2} \in \mathcal{D}$ at each ω, or equivalently:

$$\min(\bar{\sigma}(S_{m1}(j\omega)), \bar{\sigma}(S_{m2}(j\omega))) \leq 1 \quad \forall\omega \tag{8}$$

The use of similar logical combinations permits the derivation of a coupled stability condition for any class of environments that can be described as a region bounded by circular arcs and straight lines. Such regions may even be frequency dependent.

When interaction between physical systems occurs at several ports simultaneously, it may not be possible to describe the class of environments with an individual Nyquist plane. In particular, this is the case when the environment can be considered two or more separate, non-interacting systems, as shown in Figure 3a. Such an environment is termed "structured," and requires a theoretical tool beyond the Small Gain Theorem. The appropriate tool is the Small μ Theorem, and is based on the "structured singular value" (μ) introduced by Doyle [7]. The "two-block" version of the theorem provides a coupled stability criterion for environments having the structure shown in Figure 3b, and satisfying $\|S_e\|_\infty \leq 1$. The condition is given in Equation 9. Further details may be found in [5].

$$\sup_{\omega}\left(\inf_{\alpha>0}\bar{\sigma}\begin{bmatrix} S_{m11} & \alpha S_{m12} \\ \frac{1}{\alpha}S_{m21} & S_{m22}\end{bmatrix}\right) \leq 1 \tag{9}$$

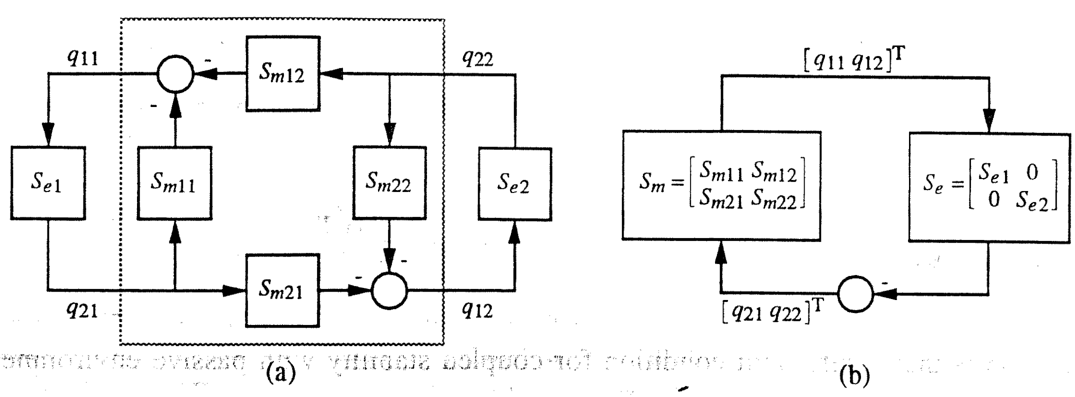


Figure 3. Equivalent representations of two-port structured interaction.

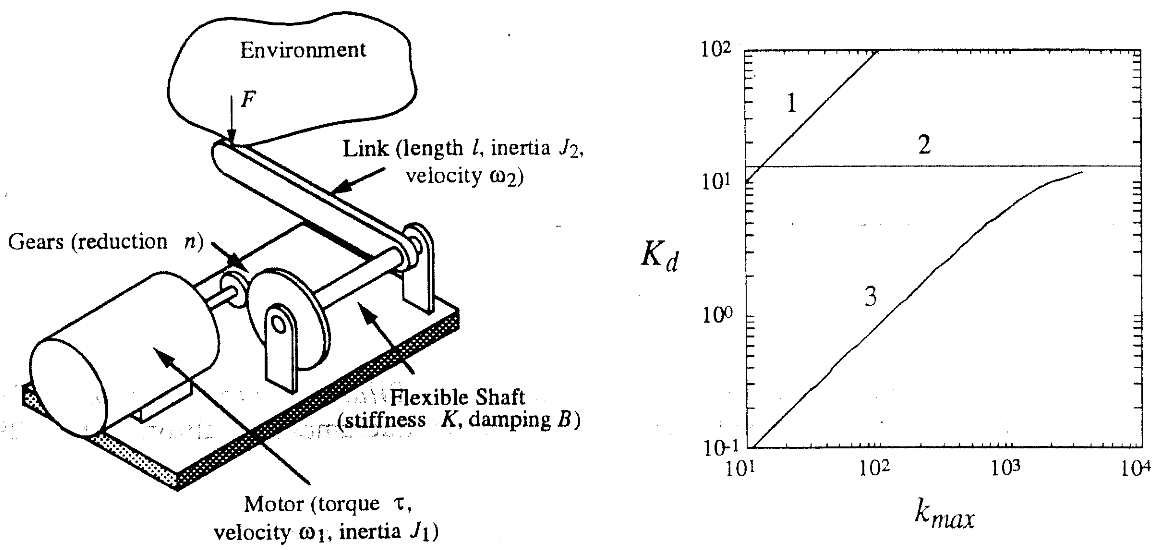
6. APPLICATION: ROBUST COMPLIANCE CONTROL

Figure 4a is a schematic model of a robot link. This model incorporates several characteristics common in robots, including gear reduction and joint flexibility. An effect of the gear reduction is to make the link nonbackdriveable, i.e., to increase its admittance. When interaction with the physical world is required — in parts assembly, for instance — high admittance is beneficial. Therefore, force feedback controllers are often used to amplify admittance. The basic idea is to close an inner, collocated servo loop (standard in industrial robots), and an outer force feedback loop which attempts to make the robot's admittance mimic the desired admittance Y_d [3].

For the sake of example, model and servo controller parameters are chosen, and the desired admittance is assumed to take the form:

$$Y_d(s) = \frac{s}{K_d}\left(\frac{1}{s^2 + 1.414s + 1}\right) \tag{10}$$

where K_d is an adjustable parameter that scales Y_d up or down. Assuming the class of environments consists of springs of stiffness $0 \leq k_e \leq k_{max}$, it is asked what minimum K_d can be achieved while ensuring coupled stability by a particular criterion. Three criteria are considered: (1) $\|Y_m W\|_\infty < 1$; (2) $\|S_m\|_\infty < 1$; and (3) $\min\{|S_m(j\omega)|, |B_m(j\omega)|\} < 1 \ \forall\ \omega$ where B_m is found as follows. The set of environment Nyquist plots is bounded, pointwise in frequency, by a disk tangent to the real axis with its center at $-jk_{max}/2\omega$. A coordinate change is found that will map this disk to $\mathcal{D}$, and then a complementary coordinate change (Equation 6) is found that maps $Y_m \rightarrow B_m$. The first criterion, which permits negative stiffness springs, is generally the most conservative, requiring $K_d > k_{max}$. The second method ignores all but the passivity of the environment, and requires $K_d >$ constant (the use of passivity is explored more deeply in [3]). The final method incorporates both passivity and the magnitude of the impedance, and produces by far the least conservative bound, as illustrated in Figure 4b.



(a) (b)

Figure 4. (a) Single link robot model. (b) Minimum K_d versus maximum stiffness of environment according to three different criteria. At any value of k_{max}, the least conservative criterion is that for which K_d is minimum.

7. APPLICATION: POWER AMPLIFICATION IN TELEMANIPULATION

A telemanipulator must interact with both a human operator and a work environment, which can collectively be considered a structured environment, as in Figure 4. If both the human and work environment are known only to be passive, coupled stability may be guaranteed by Equation 9 (Small μ Theorem), where S_m is the telemanipulator scattering matrix. This criterion is less conservative than that of passivity; indeed, it permits power amplification which is often useful in telemanipulation. For instance, consider the simplest model of a power-scaling telemanipulator, a two-port amplifier (expressed in terms of both effort-flow and scattering variables):

$$\begin{bmatrix}\varepsilon_1\\ \phi_2\end{bmatrix}=\begin{bmatrix}0 & k_\varepsilon\\ k_\phi & 0\end{bmatrix}\begin{bmatrix}\phi_1\\ \varepsilon_2\end{bmatrix};\quad \begin{bmatrix}p_{1out}\\ p_{2out}\end{bmatrix}=\frac{1}{k_\phi k_\varepsilon+1}\begin{bmatrix}k_\phi k_\varepsilon-1 & 2k_\varepsilon\\ 2k_\phi & k_\phi k_\varepsilon-1\end{bmatrix}\begin{bmatrix}p_{1in}\\ p_{2in}\end{bmatrix} \quad (11)$$

where k_ε is a force gain and k_ϕ is a velocity gain. Power output at port 2 is related to that input at port 1 by: $\varepsilon_2\phi_2 = (k_\phi/k_\varepsilon)\varepsilon_1\phi_1$. It is clear that the amplifier is not passive unless $k_\phi/k_\varepsilon=1$ (in which case it behaves as a transformer); yet Equation 9 is satisfied with $\alpha = (k_\phi/k_\varepsilon)^{1/2}$.

More realistic telemanipulator models, featuring master and slave dynamics and multiple d.o.f., may also guarantee coupled stability while providing power amplification. If, however, the bilateral controller enforces a position relationship between master and slave, the power scalings for all degrees of freedom must be the same [1]. If the bilateral controller enforces only a velocity relationship, the power scalings may be different [4].

In the future, the use of logical combination techniques along with the Small μ Theorem may lead to progressively less conservative coupled stability criteria for telemanipulators.

8. ACKNOWLEDGMENTS

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9. REFERENCES

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