Proceedings of the 1999 IEEE
International Conference on Robotics & Automation
Detroit, Michigan • May 1999

Passive Implementation for a Class of Static Nonlinear Environments in Haptic Display

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Abstract

This paper derives conditions for the absence of oscillations for a parameterized class of nonlinear environments. This class includes discrete-time environments that can exhibit non-passive behavior. The motivation for considering non-passive discrete-time environments is based on the fact that an interesting class of passive continuous-time environments have non-passive discrete-time counterparts.

A design methodology is introduced that provides relationships between the haptic device, virtual coupling and maximum negative stiffness exhibited by the environment.

1 Introduction

Envision for a moment that you are a virtual environment design engineer for haptic systems. Your job is to develop virtual perceptions for human users, with the most important design criteria being human safety. You ask yourself, what can I expect to render that satisfies this criteria? Obviously, unstable simulations are unacceptable, however another concern are simulations that exhibit oscillations. Controlled oscillations present a safety hazard and destroy any illusion the percept is attempting to convey. The current work derives conditions that are necessary to guarantee the absence of oscillations. An important contribution of this work is that it outlines a design methodology that allows environments that exhibit non-passive behavior. The interest in non-passive discrete-time environments is illustrated by the following example.

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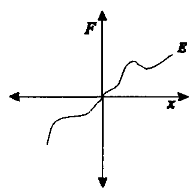

Figure 1: Force/displacement history.

Consider the force/displacement history of nonlinear stiffness (Figure 1).

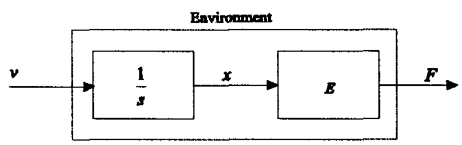

Figure 2: Continuous-time block diagram representing nonlinear stiffness. The environment block consists of an integrator and static nonlinear operator E.

In continuous-time, the block diagram representing this environment is shown in Figure (2). The continuous-time representation results in a passive environment. Next, consider a discrete-time version:

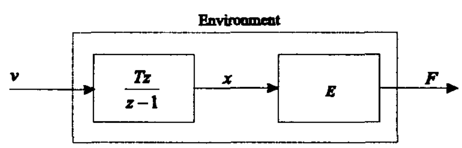

Figure 3: Discrete-time block diagram representing nonlinear stiffness. The environment block consists of an integrator and static nonlinear operator E.

Unless E is restricted to be monotonically increasing,

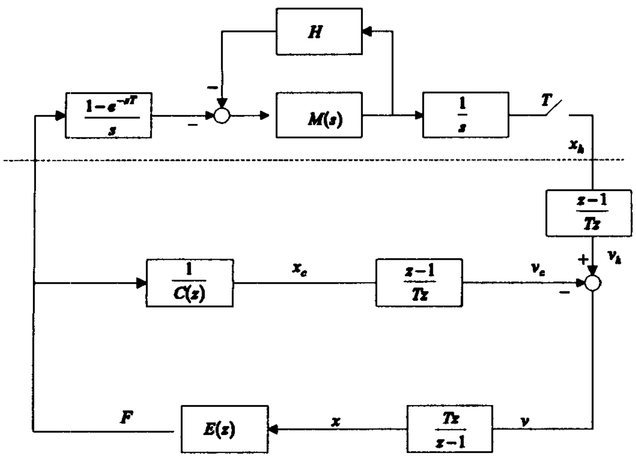

Figure 4: Block diagram for a one DOF haptic display.

the environment block is non-passive.

It is important to note that the loss of passivity is not a physical feature of the environment, rather a byproduct when obtaining the discrete-time approximation. Therefore, it seems plausible that careful design of the system should allow these discrete-time approximations to be displayed passively.

2 Previous Work

Previous work has made the assumption that the operator and virtual environment are passive, focusing on other aspects of the system to ensure stable haptic interaction. Work by Adams et al. has presented a comprehensive approach to virtual coupling design for all possible network causality. The approach taken in the current work is closely related to the results presented in [3]. These results are briefly summarized.

Consider a one degree-of-freedom (DOF) haptic display pictured in Figure (4) with operator H, haptic device M, virtual coupling C and environment E.

To achieve the goal of deriving conditions under which the haptic display appears passive, as seen by the operator, a multiplier was added to the block diagram. Using this interpretation, along with the assumption that the operator and environment are passive, a condition was placed on the amount of physical damping necessary in the device.

The current work also makes the assumption that the operator must behave in a passive manner, however we demonstrate that it is not necessary that the discrete-time environment be passive.

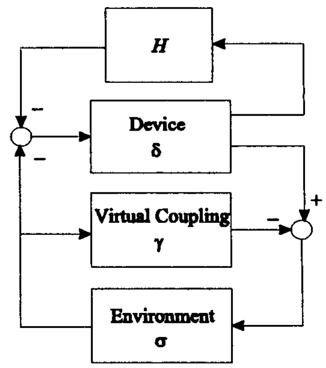

Figure 5: Graphical representation of the analysis.

Several steps in the analysis require the nyquist plot of a transfer function to be located in a particular region. A transfer function is positive real (PR) or passive if the nyquist plot lies in the closed right half plane. A transfer function is μ output strictly passive (μ-OSP) if the nyquist plot is located in a disk of finite radius.

3 Problem Statement

The remainder of this paper will derive conditions for stable haptic interaction in the sense that it guarantees the absence of oscillations.

A graphical interpretation of this argument is presented in Figure (5). Each component in the haptic structure has an associated parameter that will be used in the subsequent text to derive conditions that must be satisfied to achieve stability. The main result of this paper is that if the condition

$$\delta > \frac{\sigma T}{2} \tag{1}$$

is satisfied, and the virtual coupling is designed such that

$$\gamma = \frac{\delta\alpha}{\delta - \alpha} \tag{2}$$

holds, then a class of nonlinear non-passive environments can be displayed stably, in the sense described above, for all passive H. Equation (1) can be interpreted and used in a variety of ways.

- For a particular device parameterized by δ and fixed sample rate, a class of nonlinear environments can be identified.

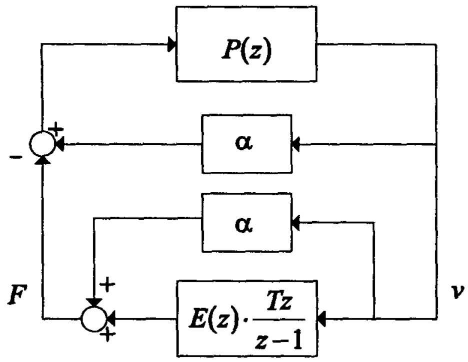

Figure 6: Coupled stability block diagram.

- For a particular device parameterized by δ and desired class of environments, a suitable sample time can be calculated.
- To guide hardware design, for a class of environments and fixed sample time, the amount of necessary physical damping can be determined.

Methods to determine values for parameters δ, γ and σ will be addressed in Section 4.

We begin by making some modifications to the coupled stability block diagram (Figure 6).

The parameter α is added to measure the amount of *excess* passivity in the top sub-system, referred to as the operator/device sub-system. It will be used to provide a connection between the physical characteristics of the haptic device (δ) and conditions imposed on the environment (σ). This results in the coupled stability block diagram shown in Figure (6). In this form the following relationship is defined

$$R(z) = \frac{P(z)}{1 - \alpha P(z)} \tag{3}$$

where $P(z)$ represents the $\mathcal{ZOH}$ equivalent of the continuous-time portion of Figure (4). Define

$$N(s) = \frac{M(s)}{1 + M(s)H(s)} \tag{4}$$

$$\begin{aligned} G(z) &= \frac{z-1}{Tz}\mathcal{ZOH}\left\{\frac{1}{s} \cdot N(s)\right\} \\ &= \frac{(z-1)^2}{Tz^2}\mathcal{Z}\left\{\frac{1}{s^2} \cdot N(s)\right\} \\ &= \frac{z-1}{Tz}\mathcal{Z}\left\{\frac{1-e^{-sT}}{s^2} \cdot N(s)\right\} \end{aligned} \tag{5}$$

$$P(z) = G(z) + \frac{z-1}{Tz} \cdot \frac{1}{C(z)} \tag{6}$$

4 Storage Function

This section will develop a storage function for use with the discrete-time version of LaSalle's invariance principle [7], to guarantee the absence of oscillations. This will be accomplished by considering the operator/device and nonlinear sub-systems separately. The positive real lemma guarantees the existence of a storage function in the operator/device sub-system, while a candidate storage function is proposed for the nonlinear sub-system. The desired storage function is obtained by adding these two functions.

4.1 Operator/Device Sub-system

The operator/device sub-system includes the haptic device, sample and hold operation and the virtual coupling network. The signal being sampled is position to accurately reflect the fact that encoders, collocated with the motors, are the only sensors present in the system. Along with obtaining a valid storage function, the main contributions of this section are as follows:

- General device model is accommodated that will allow us to characterize the haptic device with a single parameter, δ, which will be a function of device damping.
- A single transfer function (Q), independent of H, subject to a passivity condition will allow us to design the virtual coupling such that $R \in$ PR.
- The parameter α provides us with a measure of the excessive passivity possessed by the operator/device sub-system.

The goal of this section is to show that $R(z) \in$ PR, then from the positive real lemma a valid storage function, $V_L(\chi)$, exists. We begin with the ascertain that if $M(s)$ is δ-OSP, for some $\delta > 0$, and $H \in$ PR, then

$$G(e^{j\omega T}) \in \frac{1}{\delta}e^{-j\omega T} \cdot D \tag{7}$$

where $D = \{x \in \mathbb{C} \; : \; |2x - 1| \leq 1\}$, a unit disk centered at $\frac{1}{2}$, is defined for convenience. Adding α in positive feedback, along with the δ-OSP condition on $M(s)$, requires the Nyquist plot of $zG(z)$ to lie within a disk intersecting the origin and a finite positive point $\frac{1}{\delta}$.

One difficulty in analyzing this sub-system is related to the fact that every transfer function that includes H really represents a class of transfer functions

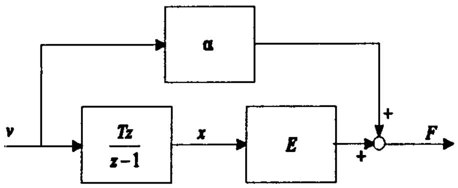

Figure 7: Environment block with parameter α. The mapping from velocity to force must be passive.

defined to include each passive H. To eliminate this difficulty we define the following expression,

$$Q(z) = \frac{z-1}{Tz}\left[\frac{1}{C(z)} - \frac{T}{2\delta}\right] \tag{8}$$

with (7) allowing it to be expressed independent of H, and require it to be γ-OSP. The utility of $Q(z)$ is that we are able to design the virtual coupling, and plot the Nyquist curve of $Q(z)$ to find γ. Then, if $M(s)$ is δ-OSP with $\delta > \alpha$ we can compute

$$\alpha = \frac{\gamma\delta}{\gamma+\delta} \tag{9}$$

Satisfying this relationship states that $R \in$ PR for every $H \in$ PR. It follows from the positive real lemma that a storage function, $V_L(\chi)$, exists that is quadratic in the states.

The next section takes α and derives a relationship with the environment parameter σ to identify a set of nonlinear environments that can be displayed passively.

4.2 Nonlinear Sub-system

This section is concerned with the block diagram in Figure (7). We demonstrate that it is possible to obtain a storage function for a class of nonlinear environments that exhibit non-passive behavior. Furthermore, a condition is derived that allows us to determine how non-passive the environment can be while still achieving stability, as outlined in Section 3. A candidate storage function will be proposed for the nonlinear environment, V_E, that will be used along with V_L to define a storage function for the coupled system. Recall that α is a measure of the *excess* passivity in the operator/device sub-block and has units of damping, therefore it helps to stabilize the environment (Figure (7)). It is important to note that α is a tool, including it in the environment analysis provides us with insight on how the overall system will behave once the two sub-systems are connected. Although α is not directly implemented, the amount of damping it represents is always available from the operator/device sub-system.

We begin the analysis by deriving the state equations associated with Figure (7).

$$x_k = x_{k-1} + Tv_k \tag{10}$$
$$F_k = E(x_k) + \alpha v_k \tag{11}$$

Defining a new state $q_{k+1} = x_k$ the state equations become:

$$q_{k+1} = q_k + Tv_k \tag{12}$$
$$F_k = E(q_{k+1}) + \alpha v_k \tag{13}$$

We propose the following storage function:

$$V_E(q) = \frac{1}{T}\int_0^q E(\xi)d\xi \tag{14}$$

It will be evident in the subsequent text, to achieve the desired goals the nonlinearity must meet certain conditions.

- $\sigma \geq \sup\{\frac{E(b)-E(a)}{a-b} : a \neq b\}$, where $\sigma \geq 0$ is the magnitude of the largest negative slope of the nonlinearity, with a and b points on the curve described by E. Note, we are only concerned with negative slope, therefore if E is non-decreasing we take $\sigma = 0$.
- $V_E(q) \geq E_{min}$, where E_{min} is the minimum energy exhibited by the nonlinearity.
- The nonlinearity described by E causes $V_E(q)$ to become radially unbounded.

We now show that the storage function in (14) is valid while providing some physical insight on the conditions imposed on the nonlinearity.

The environment with static nonlinearity $E(z)$, is passive if the environment slope

$$\sigma \leq \frac{2\alpha}{T} \tag{15}$$

For discrete-time systems the passivity condition yields

$$\Delta V_E = V_E(q_{k+1}) - V_E(q_k) \tag{16}$$
$$= \frac{1}{T}\int_{q_k}^{q_{k+1}} E(\xi)d\xi \leq F_k v_k \tag{17}$$

Physical reasoning suggests that negative stiffness is a non-passive behavior. Therefore, we seek a condition that bounds ΔV_E which will eventually lead to the constraint on the environment slope.

Consider two points **a** and **b** located on the curve defined by **E**. We assume a bound on these points:

$$\sigma \leq \frac{E(b) - E(a)}{a - b} \tag{18}$$

We define the following equation that represents a line with slope $-\sigma$ passing through the point q_{k+1}.

$$L(r) = E(q_{k+1}) - \sigma(r - q_{k+1}) \tag{19}$$

The above expression will be used to bound the storage function in Equation (14).

$$\frac{1}{T}\int_{q_k}^{q_{k+1}} E(r)dr \leq \frac{1}{T}\int_{q_k}^{q_{k+1}} L(r)dr \tag{20}$$

To continue the analysis we evaluate the right hand side of (20).

$$\frac{1}{T}\int_{q_k}^{q_{k+1}} L(r)dr = F_k v_k - (\alpha - \sigma\frac{T}{2})v_k^2 \tag{21}$$

For passivity to hold the above expression must produce a result that is $\leq F_k v_k$. To achieve this, the second term on the right hand side must be positive. Equation (15) follows when solving for σ.

For the radially unbounded condition to be satisfied, the nonlinearity must cause (14) to approach infinity as $q \to \infty$.

4.3 Coupled System

In previous sections we have derived conditions under which the equation

$$V(\chi, q) = V_L(\chi) + V_E(q) \tag{22}$$

is a valid storage function. The discrete-time version of LaSalle's invariance principle [7] can be directly applied to demonstrate that in steady state the velocity input to the environment is exactly zero. Assuming that E is defined such that (14) is both bounded from below by E_{min} and radially unbounded, this storage function guarantees the compactness of the set

$$\Omega = \{V(\chi, q) \leq V(0)\} \tag{23}$$

Since V is a non-increasing function the solutions of the coupled system are bounded and $V \to c$, c being some arbitrary constant. The fact that (23) is compact proves the existence of the positive limit set L^+ that is compact and positively invariant. Furthermore, the solutions of the coupled system tend toward L^+ as $k \to \infty$ and $V(L^+) \to c$.

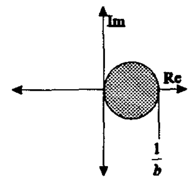

Figure 8: Nyquist region for one DOF haptic display.

Now we wish to show that $L^+ \subset \{v(k) = 0\}$. Since $V(L^+) \to c$ and L^+ is a positively invariant set, $\Delta V(L^+) = 0$. From the fact that $\Delta V \leq -\epsilon v^2(k)$ we can state:

$$\Delta V(L^+) \subset \{\Delta V = 0\} \subset \{v(k) = 0\} \tag{24}$$

It follows that the solutions tend toward $L^+ \subset \{v(k) = 0\}$.

To prove that $F(k)$ approaches a constant, we use the fact that $v = 0$ in steady state. The projection of this set for the following relationship

$$x = \frac{Tz}{z-1}v \tag{25}$$

results in a constant x. It follows that $E(x)$ is a constant in steady state.

Recall that this is achieved without requiring that the environment be passive, thus providing the designer with a larger class of environments that can be rendered while addressing human safety.

5 Results

Application of this design methodology will be illustrated using a one DOF haptic display (Figure (4)) assumed to behave like a rigid body (m) with viscous friction (b). A spring/damper virtual coupling is considered along with fixed sample time T.

$$C(z) = \frac{(KT + B)z - B}{Tz} \tag{26}$$

$$M(s) = \frac{1}{ms + b} \tag{27}$$

The task is, for a given level of physical damping, to design a virtual coupling to obtain the largest class of discrete-time nonlinear environments, based on our parameterization, that can be displayed passively. Assuming a passive H we first draw the Nyquist region of the device/human feedback loop (Figure 8).

From the Nyquist plot we observe that the disk has a radius of $\frac{1}{2b}$ and is centered at $\frac{1}{2b}$, therefore we conclude that $\delta = b$. We begin with (1) which immediately tells us the maximum negative stiffness allowed in the environment. Next, we use (15) to determine how much excess passivity is available.

To design the virtual coupling we use (8) and the fact that it must be γ-OSP. Recall,

$$Q = \frac{z-1}{Tz}\left[\frac{1}{C(z)} - \frac{T}{2b}\right] \quad (28)$$

$$= \frac{z-1}{Tz}\left[\frac{Tz}{(KT+B)z - B} - \frac{T}{2b}\right] \quad (29)$$

For choices of K and B the following inequality must hold to ensure that Q is γ-OSP.

$$\frac{(KT)^2}{2b} < KT + 2B \leq 2b \quad (30)$$

Lets take a step back and think about the role we want the virtual coupling to take. Ideally, we would like to eliminate the virtual coupling because it limits the stiffness of the environment. Unfortunately, when it is removed no stability result can be achieved. To minimize the effect it has on the environment we want to minimize the magnitude of the virtual coupling transfer function $\frac{1}{C(z)}$ over the entire range of frequencies. From the bounds on the virtual coupling parameters (30) , this is achieved by setting the virtual damping $B = 0$, and choosing $K = \frac{2b}{T}$. From this $Q \equiv 0$ making it γ-OSP for any $\gamma > 0$. If virtual damping is desired in the coupling the analysis would use (9) to solve for γ. Then the values of K and B need to be determined such that Q is γ-OSP.

6 Summary and Conclusions

This work has developed a design methodology that allows passive implementation for a class of non-passive discrete nonlinear environments. The motivation for interest in non-passive discrete-time environments was laid out in the introductory section.

Although this document targeted nonlinear environments the majority of the analysis has wide-spread application. Using the operator/device analysis to produce values for δ and α has given us stability results for linear environments with computational delay. We have also been successful in adding nonlinear damping to the environment block, which adds another condition to the nonlinear sub-system analysis restricting the environment damping to a sector.

Finally, the analysis has made it unnecessary to approximate the device behavior using a model (even though this was done in Section 5). We have plans to determine the parameter δ experimentally, which is all that is needed to proceed with the development.

Acknowledgments

Research supported in part by NSF Grant ECS-9703294.

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