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Environment Delay in Haptic Systems

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Abstract

This paper will investigate the influence environment delay has on haptic systems. Work presented in [9] and [11] demonstrated that it is possible to account for environment delay (even though it is a non-passive behavior) and still achieve a stability result based on passivity concepts. The details were given for haptic systems with environments having impedance causality. This paper will present the companion results for haptic systems with environments having admittance causality, resulting in a slightly more complicated analysis. Although the main focus will be on delay, the approach is valid to analyze any environment that exhibits non-passive behavior.

I. Introduction

Design of haptic systems that guarantees stable interaction is a difficult task that has been investigated by several researchers [1][3][2]. If the computation time needed to evolve the simulation exceeds the sample-period the simulation may unexpectedly terminate or the device may enter a region of instability. Two approaches to satisfying the real-time constraint are to increase the sample time or add delay. Increasing the sample time is usually not advised since issues of stability become more prevalent [12].

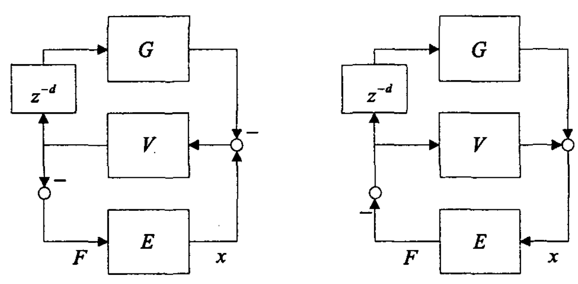

Fig. 1. Admittance and Impedance implementation block diagrams.

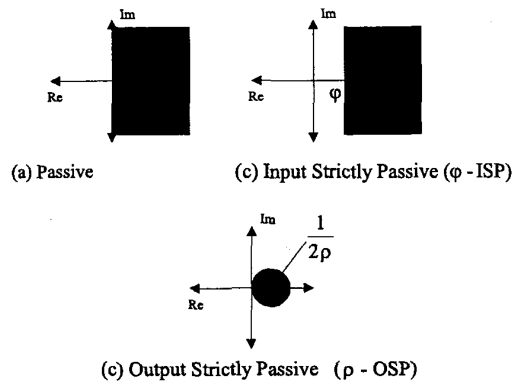

Fig. 2. Frequency domain interpretations of the different levels of passivity.

Delay in the system can be in the form of environment delay or computation delay. Environment delay is located in the environment block E (Figure 1), and used to obtain an explicit feedback loop with the virtual coupling. On the other hand, computational delay is located in the path back to the human/device block G (z^{-d}). A delay in the virtual coupling (V) is not desirable because in general we rely on the passivity characteristics of the virtual coupling to aid in displaying non-passive environments.

An important issue during implementation is environment design. If the environment model is highly nonlinear, a non-delayed implementation will require the solution of an implicit equation (that may require iteration) - a task typically not suitable for real-time implementation. The addition of a single time-step of delay avoids this situation, making the solution a function of previous values and measured inputs. The remainder of the document will investigate the effect environment delay has on stability for a class of spring/damper environments in both impedance and admittance causality[1].

[1] Admittance environments accept force and output position, whereas impedance environments accept position and output force.

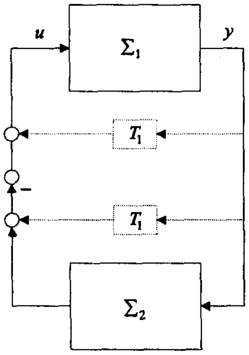

Fig. 3. Loop transformation of Coupled System.

II. Passivity and Loop Transformations

Passivity theory provides a powerful way to describe dynamically coupled systems by focusing on energy transfer [13][8]. Different levels of passivity exist, and components that contain an *excess* of passivity can be used to compensate for components that exhibit a lack of passivity. The remainder of this section will introduce passivity concepts using the storage function definition.

Definition 1: A system with input u and output y is *passive* if a non-negative function W (called a storage function) exists, that is a function of the states (x), such that

$$W(x(t)) \leq W(x(0)) + \int_0^t y(\tau)u(\tau)d\tau - \rho\int_0^t y^2(\tau)d\tau \quad (1)$$
$$-\varphi\int_0^t u^2(\tau)d\tau, \quad \forall x \in R^n, x, t \geq 0$$

with $\rho = \varphi = 0$.
The frequency domain interpretation is that the Nyquist plot lies in the closed right half plane (RHP) (Figure 2).

Definition 2: A system with input u and output y is output strictly passive (OSP) if (1) is satisfied with $\varphi = 0$ and $\rho > 0$.
Output strict passivity (OSP) is a stronger form of passivity. The frequency domain interpretation is that the Nyquist plot lies in a disk of radius $\frac{1}{2\rho}$ (Figure 2).

Definition 3: A system with input u and output y is input strictly passive (ISP) if (1) is satisfied with $\rho = 0$ and $\varphi > 0$.
Input strict passivity (ISP) is a stronger form of passivity containing an excess equal to an amount α. The frequency domain interpretation is that the Nyquist plot lies to the right of the vertical line at φ in the RHP (Figure 2).

A mapping will be referred to as ρ-OSP or φ-ISP with passivity levels ρ and φ, respectively. A mapping is referred to as having a lack of ISP or OSP if passivity (or strict passivity) can be achieved through a parallel or feedback connection of a static gain, respectively. The discrete-time counterparts of the above definitions are defined by replacing the integral with a summation [6]. The conditions on the Nyquist plots remain the same. If we define $\Delta W(k) = W(k+1) - W(k)$, we can obtain the incremental version of (1):

$$\Delta W(k) \leq y(k)u(k) - \delta y^2(k) - \varphi u^k(k) \quad (2)$$

If a component is described as having a lack of OSP or ISP the corresponding passivity level is negative, producing an additive term on the right hand side of (2). This results in energy producing behavior that must be countered by a component with an excess of the appropriate passivity level so that the energy that was generated can be dissipated. For example, consider the coupled system in Figure 3. Σ_1 is δ-OSP with storage function:

$$\Delta W_1(k) \leq y(k)u(k) - \delta y^2(k) \quad (3)$$

Σ_2 has a lack of φ-ISP with storage function

$$\Delta W_2(k) \leq -u(k)y(k) + \varphi y^2(k) \quad (4)$$

Proposing the energy function $W = W_1 + W_2$, the incremental form for the coupled system becomes:

$$\Delta W(k) = \Delta W_1(k) + \Delta W_2(k) \leq -(\delta - \varphi)y^2(k) \quad (5)$$

For the coupled system to be OSP, $\varphi < \delta$.

The amount of excess passivity available from Σ_1 can be expressed through a loop transformation, which indicates that Σ_2 can have a lack of ISP of an amount less than T_1. For example, if Σ_1 exhibits excess passivity in the form of OSP of level δ, and Σ_2 has a lack of ISP of level φ, then $\varphi < T_1 < \delta$ for the system to be stable. It is important to note that T_1 can be a constant or transfer function, depending on the desired passivity result. Loop transformations of this type will be used throughout the remainder of the paper to gain insight on how excess passivity (damping) from one component in the haptic system can be utilized by others.

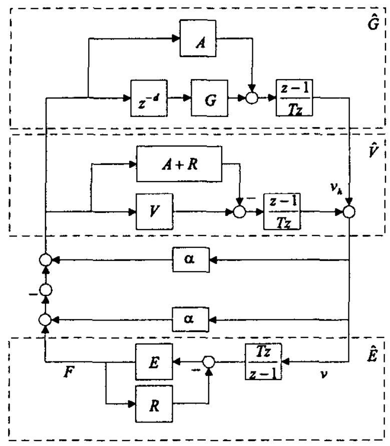

Fig. 4. Transformed haptic system with impedance environment.

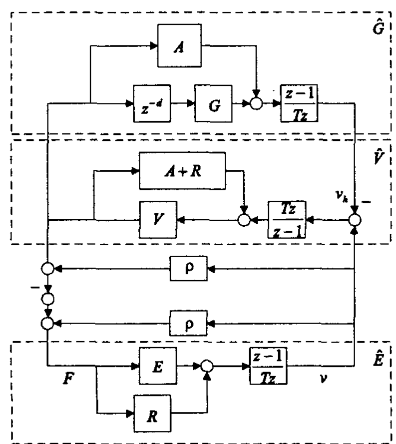

Fig. 5. Transformed haptic system with admittance environment.

III. Problem Statement

The haptic system considered in this work has four major components - the human operator, haptic device, virtual coupling and virtual environment. The human (H) and haptic device (D) are continuous-time components while the virtual coupling (V) and virtual environment (E) are discrete-time components. G represents the ZOH equivalent of the human/device feedback loop and z^{-d} denotes computational delay of d time-steps. It will be assumed that the haptic device has encoders collocated with the motors so that position is sampled and sent to the virtual environment. The causality of the virtual environment may depend on the simulation. For example, if the simulation requires evolving rigid bodies in time, it is better to integrate force as opposed to differentiating position twice. The causality of the environment defines how it interconnects with the other components in the haptic system. Figure 1 compares the different implementations. We observe from the figure, for the system with an impedance environment the human/device block G is connected in parallel with virtual coupling V, and the environment (E) is connected in feedback. For the system with an admittance environment, G is in parallel with E, and the virtual coupling is connected in feedback. Results presented in [10] [5] demonstrated that performing a loop transformation made it possible to identify excess passivity (device damping) that can be utilized to simulate non-passive environments. The remainder of this section will summarize the results for the impedance case and outline the companion results for the admittance case.

The desired stability result for the overal haptic system is cyclo-passivity[7] of the haptic display (device, virtual coupling and virtual environment). If the system is linear than passivity of the display can be demonstrated[3]. Then under passive human excitation the absence of oscillations and other persistent behavior is guaranteed in the velocity signal presented to the human. The desired stability result is achieved by performing a loop transformation on the original system, to obtain an equivalent, yet more useful form.

A. Impedance Environment

For the system with impedance environment, the expressions for the transformed system (Figure 4) are as follows:

$$\hat{G} = \frac{z-1}{Tz}[Gz^{-d} + A] \tag{6}$$

$$\hat{V} = \frac{z-1}{Tz}[V - A] \tag{7}$$

$$\hat{E} = \frac{E}{1+ER}\left[\frac{Tz}{z-1}\right] \tag{8}$$

$$A = \frac{T}{2\delta}\frac{z^d + z^{d-1} + \cdots + z + 1}{z^d} \tag{9}$$

The transformed components must satisfy the following conditions:

- $\hat{G}$ must be δ-OSP, where δ represents a damping characteristic of the device.
- $\hat{V}$ must be γ-OSP.
- $\hat{E} + \alpha$ must be passive. $\frac{E}{1+ER}$ can exhibit a lack of ISP of an amount α.

The last term showing up in both (6) and (7) is added to G to achieve a (discrete-time) δ-OSP block once discretization, computational delay and the sample/hold operator are included. So as not to change the dynamics of the original system, this term must be subtracted from the virtual coupling (see details in [5]).

The parallel connection of transformed blocks $\hat{G}$ (δ-OSP) and $\hat{V}$ (γ-OSP) is analogous to resistors in parallel, producing a block that is $\frac{\delta\gamma}{\delta+\gamma}$-OSP. Consider a storage function, W_1, for the parallel connection of $\hat{G}$ and $\hat{V}$. Since this connection is $\frac{\delta\gamma}{\delta+\gamma}$-OSP:

$$\Delta W_1 \le \frac{-\delta\gamma}{\delta+\gamma}v^2 - Fv \tag{10}$$

Consider a storage function, W_2, for the transformed environment block $\hat{E}$ that exhibits a lack of ISP of an amount α.

$$\Delta W_2 \le \alpha v^2 + Fv \tag{11}$$

Proposing an energy function $W_I = W_1 + W_2$,

$$\Delta W_I \le \left(\alpha - \frac{\delta\gamma}{\delta+\gamma}\right)v^2 \tag{12}$$

Our stability result requires ΔW_I to be decreasing, therefore

$$\alpha < \frac{\delta\gamma}{\delta+\gamma} \tag{13}$$

For $\delta, \gamma > 0$ it follows from (13) that

$$\alpha < \delta \tag{14}$$

For $\hat{G}$ to be (discrete-time) δ-OSP device D must be (continuous-time) δ-OSP, which is satisfied for a wide range of devices including ones modeled in Lagrangian form with joint flexibility and motor dynamics. In designing the virtual coupling it is desirable to maximize γ, thus maximizing α corresponding to a large class of environments. For the commonly used backwards difference spring/damper virtual coupling, the maximum amount of achievable γ is equal to δ. This implies that $\alpha \le \frac{\delta}{2}$. From (7) if $V = A = \frac{T}{2\delta}\frac{z^d+z^{d-1}+\cdots+z+1}{z^d}$ then $\hat{V} = 0$ corresponding to $\gamma = \infty$. Therefore, the virtual coupling can be designed to achieve an arbitrarily large γ, however it is unclear what type of perception it will provide. Design of the virtual coupling has proven to be an interesting problem in its own right, and will be considered in future work.

B. Admittance Environment

For the system with admittance environment the expressions for the transformed blocks are as follows (Figure 5):

$$\hat{G} = \frac{z-1}{Tz}[Gz^{-d} + A] \tag{15}$$

$$\hat{V} = \left[\frac{V}{(1 - V(A+R))}\right]\frac{Tz}{z-1} \tag{16}$$

$$\hat{E} = \frac{z-1}{Tz}[E + R] \tag{17}$$

$$A = \frac{T}{2\delta}\frac{z^d + z^{n-1} + \cdots + z + 1}{z^d} \tag{18}$$

The transformed components must satisfy the following conditions:

- $\hat{G}$ is characterized with OSP level δ representing a damping characteristic of the device.
- $\hat{V}$ is characterized with ISP level γ.
- $\frac{\hat{E}}{1+\rho\hat{E}}$ must be passive. $E + R$ can exhibit a lack of OSP of an amount ρ.

Consider a storage function W_3 for the feedback connection between the device and virtual coupling. Since $\hat{G}$ is δ-OSP and $\hat{V}$ is γ-ISP:

$$\Delta W_3 \le Fv - \left(\frac{\delta\gamma}{\delta+\gamma}\right)v^2 \tag{19}$$

The transformed environment is located in feedback with this connection, therefore it can exhibit a lack of OSP. Proposing a storage function W_4,

$$\Delta W_4 \le -Fv + \rho v^2 \tag{20}$$

A valid storage function for the entire system is $W_A = W_3 + W_4$,

$$\Delta W_A \le -\left(\frac{\delta\gamma}{\delta+\gamma} - \rho\right)v^2 \tag{21}$$

For stability, ΔW_A must be decreasing therefore,

$$\rho < \frac{\delta\gamma}{\delta+\gamma} \tag{22}$$

Recall that ρ describes the lack of OSP that the environment can exhibit. For $\delta, \gamma > 0$ it follows from (22),

$$\rho < \delta \tag{23}$$

If the transformed environment is OSP, then ρ is negative and (22) is satisfied.

Consider a virtual coupling of pure stiffness ($V = K_V$) and no computation delay (d=0), $\hat{V}$ becomes:

$$\hat{V} = \frac{Tz}{z-1}\left[\frac{K_V}{1 - (\frac{T}{2\delta} + R)K_V}\right] \tag{24}$$

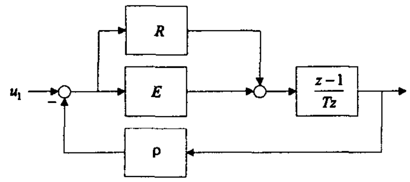

Fig. 6. Admittance environment block. The mapping from u_1 to y_1 must be passive.

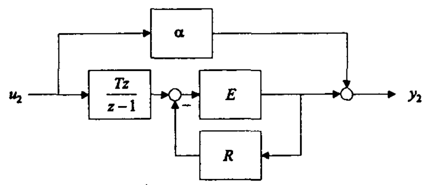

Fig. 7. Impedance environment block. The mapping from u_2 to y_2 must be passive.

To achieve a γ-ISP $\hat{V}$, the gain must be positive, therefore,

$$K_V < \frac{\frac{1}{R}\frac{2\delta}{T}}{\frac{1}{R}+\frac{2\delta}{T}} \tag{25}$$

Expression (25) implies that

$$K_V < \frac{2\delta}{T} \tag{26}$$

$$K_V < \frac{1}{R} \tag{27}$$

C. Environment Comparison

Delay has a significant impact on the steps necessary to design environments that result in stable interaction. In the impedance case, the transformation involving α reflects the amount of excess passivity exhibited by the device. In the admittance case the transformation involving R is required to resolve a lack of ISP, however it influences the design of the virtual coupling.

A result in discrete-time linear passivity theory states that if a transfer function is passive, then its reciprocal is also passive (and proper since it must have relative degree of zero). With this result, it might be tempting to assume that the inverse of an impedance environment will define an admittance environment that satisfies the passivity condition on $\hat{E}$. This is only the case when no environment delay is present. Delay is causality independent, always expressed in the form z^{-n}, therefore delayed impedance and admittance environments are no longer duals of each other. Without the inverse relationship between the environments, parameter α and R will take on their own conditions when considering a class of environments.

The following section will investigate a class of delayed linear spring/damper virtual environments. In the imedance case the device damping and virtual coupling, which produce a positive α, is sufficient to satisfy the passivity conditions (thus $R = 0$). In the admittance case, R is needed to resolve a lack of ISP, and ρ ends up being negative - therefore condition (23) is always satisfied and $\hat{E}$ is OSP.

IV. Environment Design

This section will provide the details for delayed implementation of two linear environments, relating α to the environment parameters in the impedance case and R to the environment parameters in the admittance case. We will begin with an environment of pure stiffness and then add a damper in parallel to investigate a benchmark problem in haptics - the virtual wall.

A. Delayed Stiffness

A result presented in [11] states that a passive mapping exists between u_1 and y_1 (Figure 7), with environment $E = \frac{K_E}{z}$, for an α satisfying

$$\alpha = \frac{1}{2}K_E T < \delta \tag{28}$$

This result implies that the maximum achievable environment stiffness increases with device damping δ and smaller sample-time T.

Implementation of a delayed stiffness with admittance causality, $E = \frac{1}{z}\frac{1}{K_E}$, requires

$$R = \frac{1}{K_E} \tag{29}$$

In this case the virtual environment stiffness can be anything ($K_E > 0$) because as K_E goes to infinity the effective stiffness presented to the device approaches K_V, which is bounded by $\frac{2\delta}{T}$. However, if $K_E < \frac{2\delta}{T}$ from (27) we see that K_V is limited by K_E. Therefore, the effective stiffness presented to the device will be half of the desired stiffness.

B. Delayed Virtual Wall

This section will add a damper in mechanical parallel with the spring to investigate a delayed virtual wall.

Velocity will be computed using backwards difference differentiation.

The transfer function with impedance casuality:

$$E = \frac{(K_E T + B_E)z - B_E}{Tz^2} \tag{30}$$

Results in [11] demonstrated that an α exists such that the mapping from u_2 to y_2 is passive. Passivity level α relates to environment parameters:

$$\delta > \alpha > \frac{K_E T}{2} + B_E \tag{31}$$

This result shows that maximum environment stiffness is increased as the sample-time is decreased or the physical damping characteristic of the device is increased. It also reveals that the addition of virtual damping may require a decrease in environment stiffness.

The admittance model for a delayed backwards difference virtual wall takes the form:

$$E = \frac{T}{(K_E T + B_E)z - B_E} \tag{32}$$

For the delayed virtual wall if

$$R = \frac{T}{K_E T + 2B_E} \tag{33}$$

then $\hat{E}$ is OSP. To determine the effect of adding virtual damping we must relate it back to the virtual coupling. If $K_E < \frac{2\delta}{T}$ then from (27)

$$K_V < \frac{K_E T + 2B_E}{T} \tag{34}$$

Condition (34) implies that a higher stiffness in the virtual coupling can be achieved if virtual damping is added. However, recall that K_V must also be less than $\frac{2\delta}{T}$, therefore the amount that the addition of virtual damping can enhance virtual coupling stiffness is limited by δ.

V. Discussion and Conclusions

The previous section reported results for virtual environments consisting of springs and dampers. The results indicate that when a delay is present in the environment, a higher stiffness can be achieved by implementing these environments with impedance causality. A possible conclusion is that impedance environments are better suited for this class of environments. The importance of admittance causality is clear when considering virtual environments such as rigid body simulations. Numerically differentiating position twice, which is required for impedance causality, could introduce substantial noise into the system making implementation difficult. Future work will detail the steps required to handle delayed mass elements.

One important conclusion is that the virtual coupling is affected when delay is introduced. Future work will look at an approach to automate this technique to determine the *optimal* virtual coupling based on specified performance criteria.

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