

Design of a 3R Cobot Using Continuously Variable Transmissions

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Abstract

Cobots are capable of producing virtual surfaces of high quality, using mechanical transmission elements as their basic element in place of conventional motors. Most cobots built to date have used steerable wheels as their transmission elements. We describe how continuously variable transmissions (CVTs) can be used in this capacity for a cobot with revolute joints.

The design of an "arm-like" cobot with a three-dimensional workspace is described. This cobot can implement virtual surfaces and other effects in a spherical workspace approximately 1.5 meters in diameter. Novel elements of this cobot include the use of a power disk that couples three CVTs directly.

I. Introduction

Several robotic devices have been proposed for the purpose of creating programmable constraints and virtual surfaces. One such device by Book et. al. [1], called P-TER (Passive Trajectory Enhancing Robot), is a 2-degree of freedom (dof) manipulator designed to guide its end effector along a desired path while being pushed by the user. Clutches are used to vary the coupling between the two major links of the device, while brakes are used to remove energy from the links. Delnondedieu and Troccaz [2] have developed another device, called PADyC (Passive Arm with DYnamic Constraints), intended for guided execution of potentially complex surgical strategies. The prototype system has 2 dof and uses 2 each of a motor, clutch, and free wheel to dynamically constrain each joint.

Neither of these devices is able to provide arbitrarily oriented, smooth, hard virtual surfaces.

II. Scooter Cobot

To illustrate how cobots provide smooth, hard virtual surfaces, we use the example of Scooter (shown in Fig. 1), a cobot with a three-dimensional workspace (x, y, θ) [3]. Three small motors are used to steer the wheels of which two are visible in Fig. 1. The motors cannot cause the wheels to roll; they can only change the wheels' rolling direction. A force sensor on the center post handle measures forces applied by the user.

A cobot's two simplest modes of operation are free mode (in which the user can move the cobot freely in (x,y,θ) -space) and virtual surface mode (in which only motion along a virtual constraint is allowed).

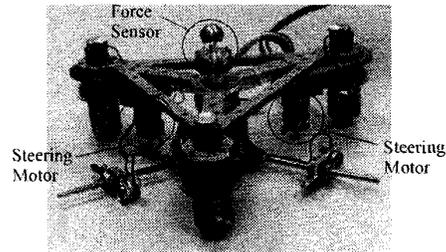


Fig. 1. Scooter three wheel cobot.

Free Mode:

In free mode Scooter operates as if it were supported by casters, like those on an office chair, which permit any desired motion direction. Unlike casters whose shafts are off center, Scooter's wheels are on straight-up shafts and are steered using motors. When the user applies a force to Scooter by pushing on the center handle, the computer monitors the force perpendicular to Scooter's rolling direction and attempts to minimize it by changing Scooter's rolling direction. Scooter's rolling direction is described by a center of rotation (COR). If the COR lies directly in the center of Scooter, the only allowed motion is rotation about the handle. If the COR is infinitely far away, corresponding to all wheel axes being parallel, then Scooter will follow a straight line. Therefore, in free mode, the computer monitors the user's forces, determines the required COR, and turns the wheels to allow that motion.

Virtual Surface Mode:

In virtual surface mode a cobot filters the user's motion. If the user brings Scooter up to a programmed virtual surface, the computer ceases to steer the wheels in a direction that minimizes the perpendicular force. Instead, the wheels are steered such that the allowed motion is tangent to the surface. The computer does continue to monitor the user-applied forces. Forces that would cause Scooter to penetrate the surface are ignored. Forces that would bring Scooter off of the surface and back into the free space are interpreted as before in free mode, and Scooter again behaves as if it were on casters.

When a cobot is in contact with a virtual surface or constraint, it is possible for the user to apply a force into the constraint that is large enough to cause the constraint to collapse. The strength of the virtual constraint is related to the mechanism by which the cobot resists

perpendicular forces. With Scooter, coulomb friction forces between the steered wheels and the working planar surface resist forces applied against the constraint. If the applied force becomes greater than the friction force, the virtual surface crumbles and the cobot enters the restricted area.

III. Rotational CVT

Scooter is restricted to a three-dimensional planar workspace because its virtual surface behavior relies on the presence of a flat working surface on which to roll. Revolute arm-like architectures have proven very versatile for robots, and so we now address the problem of creating an arm-like cobot with revolute joints.

The role of the steered wheels in Scooter is to establish a mechanically enforced ratio between the x-velocity and the y-velocity of the steering axis of each wheel. That ratio, v_y/v_x , is given by α , the steering angle of the wheel, which is under computer control. This principle may be considered obvious for a wheel, but it lies at the heart of cobots that have planar workspaces, like Scooter.

To extend cobots to workspaces typical of revolute jointed robots, we require a mechanical element whose function is analogous to that of the wheel in scooter. For revolute joints the mechanically enforced ratio is between two angular velocities, ω_1 and ω_2 , rather than two linear velocities as in scooter. Also, the ratio ω_2/ω_1 , which is enforced mechanically, must be adjustable under computer control just as the angle of each of Scooter's wheels.

The requirements above call for a continuously variable transmission, or CVT: a device which couples two angular velocities according to any adjustable ratio. Such a CVT is shown in Fig. 2.

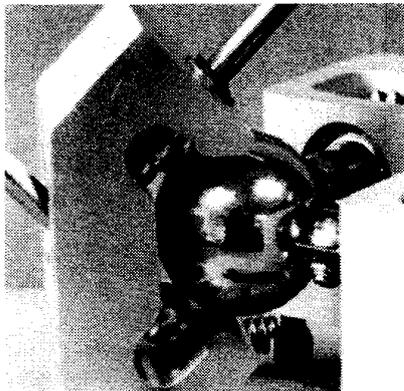


Fig. 2. Rotational CVT

A CVT holds two angular velocities in proportion: $\omega_2/\omega_1 = T$, where T is the continuously variable transmission ratio.

As diagramed in Fig. 3, the CVT consists of a sphere caged by four rollers. The rollers are arranged at the corners of a stretched tetrahedron so that the angle

subtended by the contact points is 90° (a regular tetrahedron would have angles of 108°). The two drive rollers with angular velocities ω_1 and ω_2 interface to the joints of the cobot. The other two rollers are steering rollers whose orientation controls the central sphere's rotational axis and thereby the ratio $\omega_2:\omega_1$.

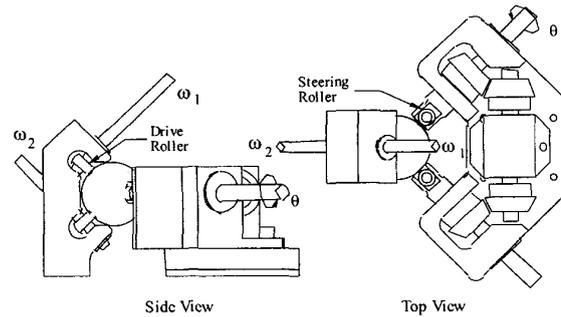


Fig. 3. Rotational CVT

Since both drive rollers are in rolling contact with the central sphere, its rotational axis must lie in the plane containing both drive rollers and pass through the non-translating center of the sphere. If the orientation of the rotational axis is located by an angle γ measured from drive roller 2, the transmission ratio T is

$$T = \tan \gamma = \frac{\omega_2}{\omega_1} \quad (1)$$

The angle γ is determined by the steering roller angle θ by

$$\gamma = \tan^{-1} \left(\frac{\sqrt{2}}{2} \tan \theta \right) - 45^\circ \quad (2)$$

The transmission ratio $\omega_2:\omega_1$ assumes the full range of values from $-\infty$ to $+\infty$ as the steering rollers are turned from -90° to $+90^\circ$ [4].

IV. Serial Cobot

Fig. 4 shows a hypothetical arm cobot that uses 1 CVT to couple its two joints. We present this diagram to show, in the simplest possible application, how a CVT is used in a revolute jointed cobot. The steering rollers along with the mechanism used to hold the CVT in place on link 1 is not shown.

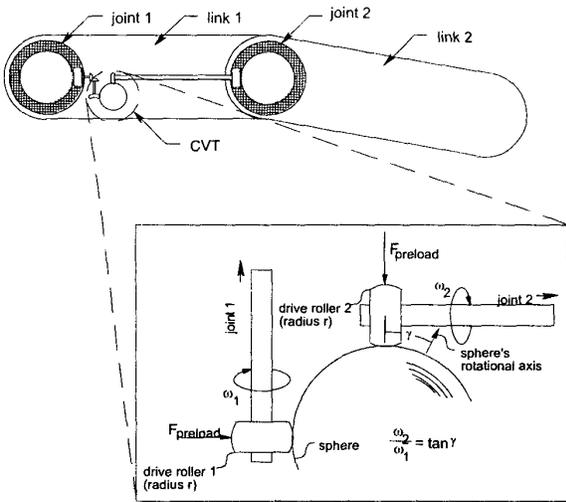


Fig. 4. Arm cobot with 2 joints

The rotation of each joint is coupled to one drive roller of the CVT. The drive rollers are held in rolling contact with the central sphere through a preload force F that is usually applied by springs. Because the ratio of drive roller angular speeds is equal to the tangent of the sphere's spin axis angle γ , the allowed endpoint velocity (v_x, v_y) in task space is also a function of γ :

$$\begin{bmatrix} v_x \\ v_y \end{bmatrix} = \mathbf{J} \begin{bmatrix} \omega_1 \\ \omega_1 \tan \gamma \end{bmatrix} \quad (3)$$

where \mathbf{J} is the cobot's Jacobian.

The equations governing free caster and constraint control are beyond the scope of this paper, but can be found in [5]. However, a basic understanding of free caster control is helpful to understanding the relationship between a CVT and the user applied endpoint forces.

The user's applied force \mathbf{F}_{ext} is read by a force sensor at the endpoint and used to assess the endpoint acceleration desired by the user. Using this desired acceleration and the endpoint velocity, it is possible to predict a desired C-space path. The principle unit normal for this path is then found and used along with the appropriate Jacobian to determine the necessary CVT steering speeds. The steering speed ω is proportional to the endpoint force perpendicular to the velocity vector F_{\perp} and inversely proportional to the endpoint speed u times the endpoint mass M or

$$\omega = \frac{F_{\perp}}{uM} \quad (4)$$

When the speed of the endpoint is zero, the steering velocity is undefined, and the steering angle is set such that the allowed motion direction is parallel to \mathbf{F}_{ext} .

A. Connecting CVTs

There are two methods of connecting CVTs to a cobot's joints. In the first method, each successive pair of joints is coupled to the drive rollers of one CVT. Fig. 5 is a schematic of a 3-joint cobot with 2 CVTs in series.

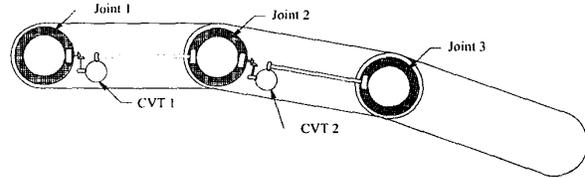


Fig. 5. 3-Link Cobot with 2 CVTs in series

The signal flow between task space velocity \mathbf{V} (v_x, v_y) and the two serially connected CVTs is diagrammed in Fig. 6. It is important to remember that the CVTs determine the joint speed ratios, while the joint speeds themselves are a function of the user applied endpoint forces.

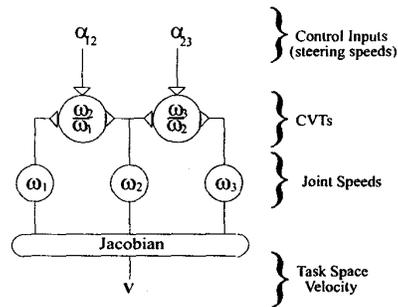


Fig. 6. Signal flow between serial CVTs

The interdependence of joint angular speeds is an important characteristic of the serial model. When an internal joint of the cobot has a near zero angular speed, an extremely high transmission ratio will be required by another CVT in the chain. Take for example the 3-joint, 2-CVT model of Fig. 5. If the desired joint angular velocities are $\omega_1 = \omega_d$, $\omega_2 \ll \omega_d$, and $\omega_3 = 2\omega_d$ the following CVT transmission ratios are required:

$$T_1 = \frac{\ll \omega_d}{\omega_d} \quad T_2 = \frac{2\omega_d}{\ll \omega_d} \quad (5)$$

The first transmission ratio is near zero and attainable. The second transmission ratio is near infinite and not possible because friction internal to the CVT bounds the maximum transmission ratio to approximately 20:1. A second issue with the serial chain model is that transmission ratio errors in one CVT are propagated to the others.

CVTs can also be connected in parallel. A three joint example is shown in Fig. 7. The angular velocity of each

joint is coupled to a separate drive roller and the remaining three drive rollers are tied to a common shaft.

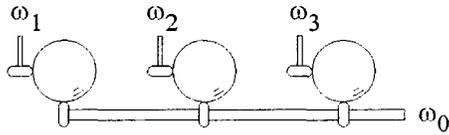


Fig. 7. CVTs connected in parallel

In the parallel configuration, a CVT transmission ratio T_i relates the joint velocity ω_i to the common shaft velocity ω_0

$$T_i = \frac{\omega_i}{\omega_0}, \quad (6)$$

where the angular velocity of the common shaft is

$$\omega_0 = \frac{\omega_1 + \omega_2 + \omega_3}{T_1 + T_2 + T_3}. \quad (7)$$

The signal flow between task space velocity \mathbf{V} and the 3 CVTs in parallel is diagrammed in Fig. 8.

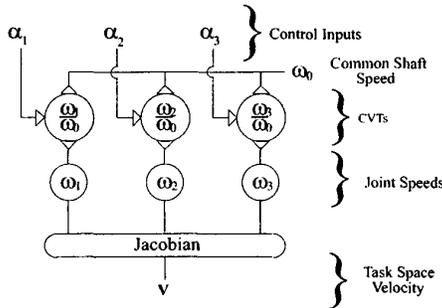


Fig. 8. Signal flow between parallel CVTs

It was shown in the serial case that a near zero angular velocity for one joint required an unattainably high transmission ratio in a neighboring CVT. In contrast, in the parallel configuration CVT, transmission ratios are proportional to the angular speed of their joints. Therefore, a near zero joint speed in the parallel configuration requires merely that its own CVT have a transmission ratio which is also nearly zero. So, the speed ratio between any two joints connected by CVTs in parallel can assume the full range of values ($-\infty$ to $+\infty$) with finite CVT transmission ratios.

Another interesting characteristic of CVTs in parallel is their ability to caster without steering. If the transmission ratios of each CVT are set to infinity, the

common shaft has zero angular velocity and each joint can rotate freely without respect to any other joint. The only other time that the speed of the common shaft is zero is when the speed of all joints is zero.

B. Adding Power

Traditionally, cobots are physically passive: their kinetic energy is bounded by the energy supplied by the user's hand. However, when we started to design cobots with larger gear ratios to create harder constraints, the increased friction reflected to the user became an issue. There was also the desire to make cobots more responsive by magnifying the user's force at low speed. The most convenient way to accomplish these goals is to add kinetic energy to the cobot using a motor.

The common shaft of CVTs in parallel provides an elegant attachment point for a power assist motor. When a motor is added to the common shaft, the signal flow changes - the angular velocity of the common shaft is now an input to each CVT (see Fig. 9).

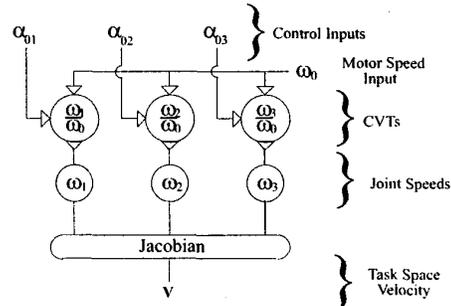


Fig. 9. Signal flow between parallel CVTs with power assist

By equation (5), a joint velocity ω_i is the product of the CVT transmission ratio T_i and the angular velocity of the common shaft ω_0 . Therefore, the cobot's task space velocity \mathbf{V} is

$$\mathbf{V} = \mathbf{J}\mathbf{T}\omega_0. \quad (8)$$

The same idea holds for an endpoint force. Each joint torque τ_i is a product of the common shaft torque τ_0 and the inverse of the transmission ratio such that the force \mathbf{F}_{xyz} reflected to the endpoint is

$$\mathbf{F}_{xyz} = \mathbf{J}^T \frac{1}{\mathbf{T}} \tau_0. \quad (9)$$

The noteworthy result here is that regardless of the dimension of the cobot's taskspace, a single motor coupled to the common shaft of CVTs in parallel produces an endpoint force and speed that is parallel to the allowed motion direction.

V. Arm Cobot with Power Assist

We have designed a cobot with a parallelogram arm configuration as shown in Fig. 10. It has a 3 dimensional (x,y,z) workspace and a 4-link parallelogram configuration which is counter-balanced for gravity. Its 3 CVTs are connected in parallel, and a power assist motor delivers power to each of the three joints through a wheel that is in rolling contact with each of the CVTs. A force sensor is located at the end effector.

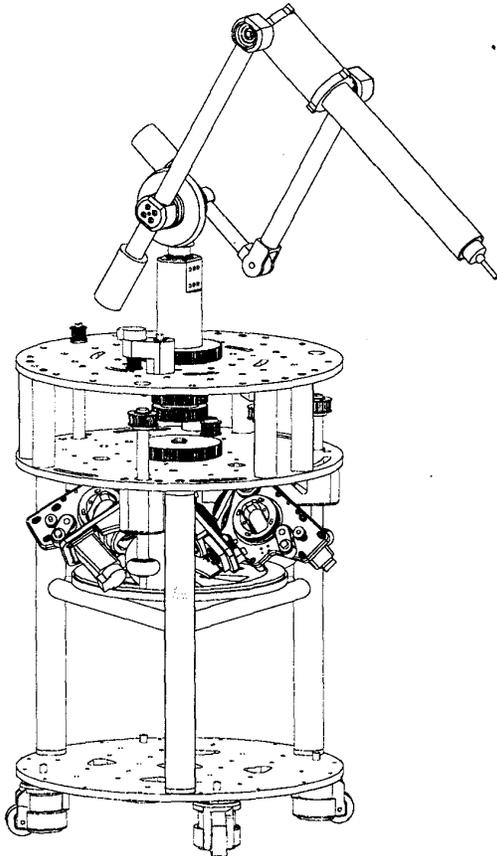


Fig. 10. Parallelogram arm cobot

It was decided early on that the cobot's CVTs would remain stationary (grounded) during motion of the arm. Connecting each CVT to ground lowers the complexity of the design and decouples the mass of the CVT subsystems from the arm's dynamics. The resulting design couples the rotation of the joints to three concentric shafts. Two sets of bevel gears connect the two non-vertical joint axes to the two innermost shafts. The third joint does not require bevel gears because its axis of rotation is already parallel to the CVTs' drive roller shafts.

Graphite was chosen for the links due to its high bending strength ($E = 96\text{MPa}$) and low weight ($\rho = 1.66\text{ g/cm}^3$). The largest diameter link has a 8.26cm OD, and the arm's reach is over 76cm. With the support stand attached, the origin of the three x, y, and z joint rotations

is approximately 1.5m above the floor. The workspace of the cobot is diagramed in Fig. 11.

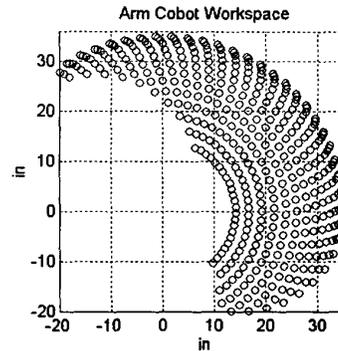


Fig. 11. Arm workspace

The links are counter-balanced against gravity for all arm configurations by two counter weights. The moments provided by the counter weights are approximately 6.6N-m and 3.8N-m.

Considering that there are three CVTs in this design, the size of the CVT dictates the footprint of the entire mechanism. The original CVT design (Fig. 3) with its long steering axes was ruled out for having too much wasted volume. The new design shown in Fig. 12 is much more compact than the original. The steering axes have been replaced with low profile steering hubs that surround the steering rollers. Equal and opposite rotation of the steering hubs is ensured by bevel gears that are synchronized to each hub through timing belts (belts are not shown). There is a 45-watt steering motor with encoder on each CVT assembly.

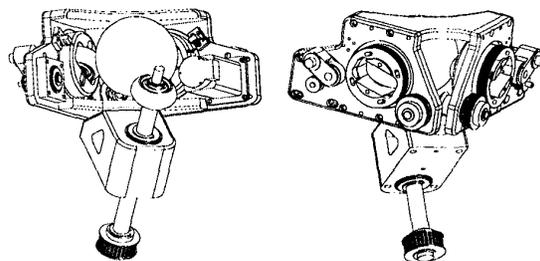


Fig. 12. Front and back view of the compact CVT

As noted earlier, CVTs in parallel have the rotation of one of their drive rollers coupled to a common shaft. The CVT of Fig. 12 has only one drive roller. In order to couple the rotations of each CVT together there is a common wheel (instead of a common shaft) that is in rolling contact with the central sphere of each CVT as shown in Fig. 13. This wheel is called the power wheel and is driven using a motor (the timing belt pulley for the motor and wheel axle is hidden on the bottom side of the support plate).

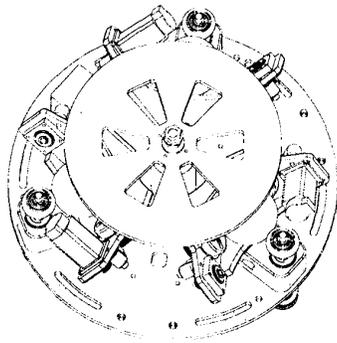


Fig. 13. 3 CVTs and a power wheel

The power wheel is made from an aluminum plate that has a rubber running surface on one side. Sections of material are removed from the wheel to lower its inertia. The power wheel has a diameter of 36.8cm, and its running surface contacts the CVTs 16.5cm from the center post.

There are many benefits to the symmetric arrangement of CVTs. It permits a single spring on the power wheel's axle to apply an equal preload force to all CVTs. The spring currently being used can apply a maximum of 22.7Kg to the rolling elements of each CVT. Also, in this arrangement, the drive roller shafts are parallel to their joint shaft axes allowing power transfer between the two with zero backlash timing belts.

The coulomb friction forces that exist between the rolling elements of the CVTs determine the force of constraint that the arm cobot can display. Taking $\mu_s = 0.8$ as the coefficient of static friction between the CVTs rolling elements, a drive roller radius of 2.84cm, and a normal force of 133.4N, the resulting maximum torque that the drive rollers can resist is 3.0N-m. With a gear ratio of 6:1 between the drive rollers and the joints, the maximum sustainable joint torques are 18.0N-m.

Fig. 14 uses force ellipses to display the static force characteristics for the arm [6]. The major (minor) axis of each ellipse represents the maximum (minimum) endpoint force that can be resisted in the direction of the axis. The largest force that can be supported at a position regardless of direction is recorded in pounds next to each ellipse.

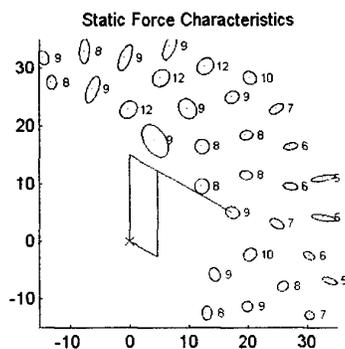


Fig. 14. Static force characteristics

VI. Contribution of Design

The parallelogram arm cobot will be the first 3R cobot. Its ability to move in traditional x,y,z 3-space opens an entire new class of tasks to cobotic solutions. The addition of power assist to the traditional passive cobotic model will result in a cobot that can reduce or magnify the inertia that is reflected to the user making larger cobots or cobots with large transmission ratios possible.

The powered arm will be able to perform tasks autonomously like a traditional robot while remaining backdrivable. A backdrivable arm is attractive to persons that want a powered manipulator that can also be easily positioned by hand, such as those interested in robot-assisted surgery.

VII. References

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