

THE CONTROL OF DYNAMICALLY INTERACTING  
SYSTEMS

by

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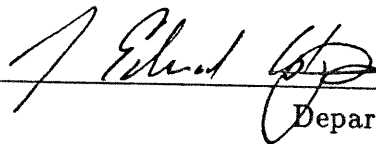
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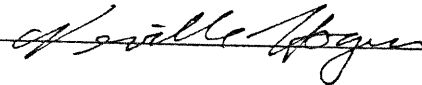
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## ABSTRACT

This thesis addresses the analysis and design of controllers for dynamically interacting systems. The example of a robotic manipulator which must interact with an arbitrary passive environment is considered in detail.

An approach to the design of *interaction controllers* is presented in terms of a hierarchical set of design specifications and is contrasted with an approach to servo design. It is shown that the emphasis on interaction creates a need for *coupled stability* and *interactive behavior* specifications.

Methods are developed for the analysis of coupled stability and interactive behavior. A necessary and sufficient condition for the stability of a linear, time-invariant plant coupled to an arbitrary passive environment is derived. An alternative test for coupled stability requiring the construction of two root loci, one representing interaction with springs, and one representing interaction with masses, is also developed.

Experiments performed to examine the utility of these methods are described. A variety of controllers were implemented on a two-link manipulator and a device for measuring the endpoint impedance of the manipulator was constructed. Impedance measurements of the closed-loop systems were made, and interaction with springs and masses was examined. These experiments indicate that a measurement of the impedance is an effective predictor of interactive behavior. In addition, these experiments demonstrate that systems with identical servo responses can exhibit significantly different interactive behaviors, and that contact instability can occur in the absence of force feedback.

The concepts of a *passive physical equivalent* and an *uncontrollable element* are introduced. These concepts are used to analyze the contact instability phenomenon associated with force feedback, and to make recommendations for improved force control.

Finally, approaches to the design of interaction controllers are presented and analyzed. The relative merits of various design methods, including "servo masking" and the "target model referenced controller", are discussed.

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To Mom and Dad

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# Nomenclature

## General Notation

$a$	scalar
$\mathbf{a}$	vector
$A$	matrix or scalar, as indicated by context
$A_c$	matrix corresponding to closed loop system
$A_s$	matrix corresponding to servo system
$A_t$	matrix corresponding to target system
$a_0$	reference value

## Acronyms

LHP	left half $s$ -plane
LTI	linear time-invariant
PD	positive definite
PSD	positive semi-definite
RHP	right half $s$ -plane
TMRC	target model referenced controller

## Mathematical Symbols

$a^*$	complex conjugate of $a$
$\mathbf{a}', A'$	transpose of $\mathbf{a}$ or $A$
$\mathbf{a}^H, A^H$	complex conjugate transpose (Hermitian) of $\mathbf{a}$ or $A$ .
$\ \mathbf{a}\ , \ \mathbf{a}\ $	Euclidean norm ( $\sqrt{\mathbf{a}^* \mathbf{a}}, \sqrt{\mathbf{a}^H \mathbf{a}}$ )
$\langle \mathbf{a}, \mathbf{b} \rangle$	inner product
$\sigma(A)$	singular value of $A$
$Re\{a\}, \Re a$	real part of $a$
$Im\{a\}, \Im a$	imaginary part of $a$
$j$	$\sqrt{-1}$

## Bond Graph Symbols

0	common effort junction
1	common flow junction
$S_e$	effort source
$S_f$	flow source
$I$	admittance causality storage (e.g., inertia, inductor)
$C$	impedance causality storage (e.g., spring, capacitor)
$R$	dissipator (e.g., damper, resistor)
$TF$	transformer
$GY$	gyrator

## Miscellaneous

$A$	state matrix
$b, B$	viscous damping coefficient
$B$	input matrix
$C$	output matrix
$C(s)$	closed loop transfer function
$D$	feedforward matrix, determinant term
$e$	effort, error
$E$	energy
$f$	flow
$F$	force
$G$	force feedback gain
$G(s)$	plant transfer function
$G_{xx}(\omega), G_{xy}(\omega), G_{xFx}(\omega), G_{xFy}(\omega)$	auto- and cross-spectral densities
$H_x(\omega), H_y(\omega)$	transfer functions from $x$ position to $x$ and $y$ forces
$I$	inertia matrix
$J$	jacobian matrix
$k, K$	stiffness
$L$	environmental input matrix
$m, M$	mass
$P$	power, PD matrix in energy function
$r$	reference
$s$	Laplace variable ( $\sigma + j\omega$ )
$S$	scattering matrix
$S(\omega)$	transfer function from $x$ position to $y$ position
$t$	time
$T$	time delay

$u, U$	control effort
$v$	velocity
$x, X, y$	positions
$Y(s)$	admittance
$Z(s)$	impedance
$\alpha$	amplification factor
$\beta$	two-link manipulator damping coefficient
$\gamma$	coherence
$\varepsilon$	error
$\omega$	imaginary part of $s$ , joint angular velocity
$\sigma$	real part of $s$
$\tau$	time constant
$\theta$	joint angle
$\Theta$	state vector of two-link manipulator