LINEAR ELECTROSTATIC ACTUATORS: GAP MAINTENANCE VIA FLUID BEARINGS

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This paper addresses a novel actuator for manufacturing applications, the “electrostatic artificial muscle.” Artificial muscle is composed of a dense array of small linear actuators. Its promise lies in the prospect of high performance (e.g., higher force-to-weight ratio and peak acceleration than a comparable magnetic motor), clean, quiet operation, and design versatility (especially the elimination of transmissions in many applications). The characteristics of artificial muscle are particularly appealing for applications in robotics and high-speed automation.

A model of a linear electrostatic induction motor is presented to illustrate the potential for high performance as well as the difficulty of “gap maintenance.” Gap maintenance refers to the demanding task of preserving a uniform, narrow gap between “slider” and stator in the presence of destabilizing electrostatic forces. A novel approach to gap maintenance, the use of dielectric fluid bearings, is presented. Analysis of a simple, 2-D motor model shows that gap maintenance and motor efficiency may be characterized by two nondimensional parameters: a levitation number, and a gap aspect ratio. It is shown that achieving both low-speed levitation and high efficiency requires long, narrow gaps (high aspect ratio). The results of this analysis are extended to a more complex model featuring an unconstrained, rigid slider. An experimental study of fluid bearings is also presented.

NOMENCLATURE

\[ \begin{align*}
  c & \quad \text{synchronous velocity (} = \omega/k) \\
  d & \quad \text{nominal gap thickness} \\
  d_i & \quad \text{nominal inlet gap thickness} \\
  d_o & \quad \text{nominal outlet gap thickness} \\
  F & \quad \text{force} \\
  k & \quad \text{wave number} \\
  K_e & \quad \text{electrostatic stiffness} \\
  K_f & \quad \text{fluid stiffness} \\
  l & \quad \text{slider length} \\
  l_o & \quad \text{length from trailing edge to step} \\
  M & \quad \text{moment} \\
  N_L & \quad \text{levitation number} \\
  N_{L} & \quad \text{synchronous levitation number} \\
  p & \quad \text{pressure} \\
  P & \quad \text{power} \\
  S & \quad \text{slip} \\
  t & \quad \text{time} \\
  t_r & \quad \text{slider thickness} \\
  v & \quad \text{slider velocity} \\
  V & \quad \text{voltage} \\
  y & \quad \text{slider displacement along y-axis} \\
  w & \quad \text{slider width} \\
  \langle x \rangle & \quad \text{time averaged value of } x \\
  \bar{x} & \quad \text{normalized value of } x \\
  \gamma & \quad \text{strain rate} \\
  \delta & \quad \text{step height} \\
  \varepsilon_0 & \quad \text{permittivity of free space} \\
  \varepsilon_f & \quad \text{fluid permittivity} \\
  \varepsilon_r & \quad \text{slider permittivity} \\
  \zeta_o & \quad l_o/l \\
  \eta & \quad \text{efficiency} \\
  \theta & \quad \text{pitch angle of slider} \\
  \mu & \quad \text{viscosity} \\
  \rho & \quad \text{fluid density} \\
  \sigma & \quad \text{stress} \\
  \sigma_f & \quad \text{fluid conductivity} \\
  \sigma_r & \quad \text{slider conductivity} \\
  \tau & \quad \text{time constant} \\
  \phi & \quad \text{roll angle of slider} \\
  \psi & \quad \text{yaw angle of slider} \\
  \Psi & \quad \text{electric potential} \\
  \omega & \quad \text{frequency of excitation}
\end{align*} \]

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1. ELECTROSTATIC MOTORS

1.1. Background

This paper addresses an ancient but little known actuator technology—electrostatic motors. Among the pioneers of electrostatic motors was Benjamin Franklin, who built his first working model in 1748.
more than a century before electromagnetic motors were invented. Yet, while electrostatic motors remained more or less a scientific curiosity, electromagnetic motors progressed rapidly to technological reality. The reasons for this are a little mystery. Electromagnetic motors offer numerous advantages over their electrostatic counterparts, but one is particularly striking: the energy density that can be sustained in a magnetic field is some four orders of magnitude greater than that which can be sustained by an electric field (under "ordinary" conditions discussed below).

Yet now, after the passage of another century, interest in electrostatic motors has once again arisen. There are two principal reasons for this rebirth. The first, and best documented, is the emerging technology of micromachining. Silicon micromachining, for instance, has been used to fabricate a wide array of rotary electrostatic motors having rotor diameters of 100 μm or less. The principal reason that these motors are electrostatic is that the generation of electric fields is consistent with the materials and geometries of micromachined devices. In addition, it has been argued that the small scale of these devices favors electrostatic actuation over electromagnetic. Recently, however, convincing counter-arguments have been made. It is worth noting that, with the advent of techniques which permit the use of ferromagnetic materials in micromachining, interest in magnetic micromotors has risen sharply.

The second reason for the rebirth is not as well known, but is of potentially greater interest to those in the fields of robotics and manufacturing. It is the development of "aggregate actuators," or "artificial muscle." As the latter term implies, such actuators attempt to emulate certain properties of biological muscle. One of the most attractive characteristics of muscle is its structure—muscle comprises an extremely dense packing of tiny, linear actuators. Essentially its entire volume is devoted to force production. As a consequence, muscle features a rather large power-to-weight ratio, it comes in endless shapes and sizes, and it is well suited to motion control.

The latter point is particularly important. Electromagnetic motors, especially small ones, tend to operate at high speeds with low torque. To drive significant loads, they require transmissions (which increase torque and may also convert rotary to linear motion). Transmissions have another property: the mechanical impedance at the output of a motor/transmission is typically much greater than that of the motor alone. This makes the motor insensitive to reaction forces from its load, a characteristic which is desirable in many instances (e.g. machine tool drives), but undesirable in many others (e.g. parts assembly). In addition, transmissions consume money and space, and limit design flexibility. By generating high forces at low speeds, muscle eliminates the need for transmissions.

Although many actuator technologies have been explored in hopes of emulating the overall characteristics of muscle, electrostatic actuation offers the further possibility of emulating the structure of muscle. The merits of electrostatic motors in this application, and the design approaches that have been explored, are reviewed in the next section.

1.2. Aggregate electrostatic actuators

The electric field strength \( E \) in an air-gap electrostatic motor is, in usual practice, limited to the theoretical strength of air \( (3 \times 10^6 \text{ V/m at atmospheric pressure}) \). This limits the air-gap energy density \( \varepsilon_0 E^2 / 2 \) to about \( 40 \text{ J/m}^3 \), or about four orders of magnitude less than the theoretical limit for electromagnetic actuation \( B^2 / 2 \mu_0 \), assuming a saturation field \( B \) on the order of \( 1 \text{ Wb/m}^2 \). As indicated above, the preeminence of magnetic motors is a little mystery. This comparison can be rather dramatically altered, however, by recognizing that electrostatic actuators need not be built in the image of conventional magnetic motors.

Most important in shifting the balance is the fact that the energy density in the gap and the energy density for the actuator as a whole are not the same quantity. In a permanent magnet motor, although almost all the energy is stored in the gap, the energy density is proportional to the square of the field strength, which is in turn proportional to the volume of the magnets. A ferromagnetic material is also needed for a return path. Thus, the volume required to generate and sustain a magnetic field is tremendously greater than the gap volume required to store energy. Electrostatic motors differ because electric field lines begin and end on charges. In a typical design, these charges reside on electrode surfaces, and a significant electric field is found only in the gap. Thus, gap volume may compose a substantial fraction of the actuator volume. In addition, the electrodes may be thin film conductors patterned on a light-weight, dielectric substrate such as a plastic film.

While close-packing is important, it will not account for four orders of magnitude unless the gaps are much smaller and packed much more closely than present technologies allow. It is also necessary to increase the energy density within the gap. Fortunately, this is possible. For instance, it is often noted that the Paschen effect allows higher field strengths to be sustained in very small gaps. In air at atmospheric pressure, however, this means gap widths of less than 4 μm, which may be very difficult to achieve over the vast surface area desirable in an artificial muscle. Another approach is to use a different material in the gap. Silicone oils, for instance, typically feature breakdown strengths greater than \( 10^8 \text{ V/m} \), in some cases approaching \( 10^9 \text{ V/m} \). As discussed in Sections 3 and 4, liquid-filled gaps may also act as fluid film bearings, helping to maintain the gap in the presence of destabilizing electric fields.

A final point favoring electrostatic motors is that they may, in principle, generate a stall force with zero energy consumption. Direct-current electromagnetic motors commonly used in motion control consume
maximum energy at stall. This is usually an important constraint in the selection of motors for robots.

When these various factors are combined, high-density electrostatic actuators begin to look rather appealing. In recent years, several attempts have been made to produce such actuators. Two basic approaches have prevailed. Actuators built by Yamaguchi et al.\textsuperscript{38} are examples of the first approach. As illustrated in Fig. 1, these actuators comprise a profusion of small cells, each of which is a deformable capacitor that flattens under the application of a voltage. A notable feature of this design is that no antifriction bearings are required. A similar "cell actuator" has been described by Bobbio et al.\textsuperscript{5} and another has been proposed by Pelrine et al.\textsuperscript{26}

Actuators built by Higuchi and coworkers\textsuperscript{9,22,23} are examples of the second approach. These "film actuators" comprise multiple layers of linear motors, each composed of a "slider" and stator (Fig. 2). These motors are usually patterned after conventional types of magnetic motors (synchronous, variable reluctance, induction, etc.). For instance, the actuator described in Ref. 22 may be classified as a three-phase, salient-pole machine.\textsuperscript{34} The actuator described in Ref. 9, as well as one developed by Matsumoto,\textsuperscript{17} is a three-phase induction machine. Power densities of up to 230 W/kg, and force per unit weight of up to 630 N/kg have been reported for a film actuator.\textsuperscript{22} These values compare quite well with state-of-the-art electromagnetic motors. The motor's peak efficiency, however, is only 22% (assuming that reactive power can be reused).

Several interesting points may be made when comparing the two approaches. Both types of motors store as much energy capacitively as they transduce mechanically. To achieve efficiencies greater than 50%, this stored energy must be reclaimed. Generally speaking, it is easier to reclaim energy if the excitation is a fixed-frequency alternating current (or voltage) than if it is a pulse train. This would favor film actuators. An added disadvantage of cell actuators is that they store energy elastically, which can also be difficult to reclaim under some loading conditions. Cell actuators may take advantage of the Paschen effect if their gaps are very tiny, but because of gap variation cannot be fluid-filled without significant losses. Film actuators can employ either air gaps or fluid-filled gaps (those described in Ref. 22 are fluid-filled).

Despite their numerous advantages, film actuators pose significant technical challenges. For instance, because the interdigitated layers of a film actuator slide past one another in operation (thus the name "slider" instead of "rotor"), it is important that the friction between them be minimized. More generally, it is important that a uniform gap be maintained between slider and stator, as this separation is usually a critical design parameter. Thus, it is necessary to develop an anti-friction bearing which can maintain a narrow (< 100 μm), uniform gap over potentially large areas (> 100 cm²) without creating local field concentrations. To make the problem more difficult, strong attractive forces of electrostatic origin generally exist between the slider and stator. These forces tend to destabilize the gap.

A major objective of this paper is to demonstrate the feasibility of fluid bearings as a solution to this problem. In the next section, a model of a film actuator is presented. The gap maintenance problem is examined in the context of this model. The model is also used to show that performance comparable to biological muscle is, in principle, attainable. In Section 3, approaches to gap maintenance are reviewed, and fluid lubrication is selected for further study. In Section 4, a model of a fluid bearing is developed and analysed, and Section 5 reports on an experiment which confirms one of the major predictions of the model. A discussion and conclusions are given in Section 6.

2. MODEL OF AN INDUCTION MOTOR; FORCE PRODUCTION AND INSTABILITY

2.1. Model and analysis

The objective of this section is to develop expressions for the motive and transverse forces developed by a linear electrostatic motor (film actuator). These results will provide a basis for the fluid bearing analysis that follows. Figure 3 is a schematic of the motor model, which consists of a slider immersed in a fluid between two stators. For the sake of an example, we take this to be an induction motor, which operates on the basis of a conductivity discontinuity at the slider–fluid interface. The analysis of this type of motor was pioneered by Melcher,\textsuperscript{20,21,35} and was first applied to micromotors

Fig. 1. Cell type actuator.

Fig. 2. Film type actuator.

Fig. 3. Schematic of linear induction motor.
by Bart and Lang.\textsuperscript{4} We make the following assumptions and approximations:

- Charge conduction in both the fluid and the slider is governed by Ohm’s law.
- Neither the fluid nor slider bulk supports free charge. In practice, this may not always hold true. Indeed electrohydrodynamic pumping based on electrical forces acting on free charges in a fluid has been demonstrated. However, free charge will occur only when the electric field at the solid–liquid interface is intense enough to cause coronal breakdown.
- There is no mechanism for charge conduction along the slider’s surface. Surface conductivity effects, however, may be easily incorporated into the analysis and do not affect the results quantitatively.
- Voltages on the two stators are traveling waves, described by

$$V(x, t) = V \cos(\omega t - kx).$$

In practice, this is approximated with multiphase electrode arrangements, and spatial harmonics will exist. Solutions may then be found by superposition.\textsuperscript{32}

- Fringing fields at the ends of the slider are neglected. This is reasonable because the gap is assumed to be very narrow relative to the length and width dimensions.
- The gaps both above and below the slider are assumed to be constant (thickness $d$).

Given the above, it is readily demonstrated that the following potential function solves Laplace’s equation in both fluid gaps and the slider:\textsuperscript{35}

$$\psi(x, y, t) = \text{Re}\{[C_1 \sinh(ky) + C_2 \cosh(ky)]e^{i(\omega t - kx)}\}.\tag{2}$$

The constants are solved from boundary conditions as detailed in Refs 4, 17, 32 and 35. From this solution, time-averaged transverse and motive forces, as well as other quantities of interest, are readily found. For instance, the (time-averaged) motive force per unit length and width at $y = 0$ is

$$\langle F^* \rangle = 0.4 \left(\frac{\varepsilon_f V^2}{d^2}\right) \frac{S}{1 + S^2}.\tag{5}$$

In the following, we will work largely with normalized equations. The force per unit length and width has the units of stress, and can be normalized by the Maxwell stress $\varepsilon_f V^2/d^2$:

$$F^* = 0.4 \frac{S}{1 + S^2}.\tag{6}$$

This force–speed characteristic is plotted in Fig. 6(b) (dashed line).

The transverse force may be found analytically also. The expression, however, is complicated and uninformative even once parameters are chosen. As will be shown, our main interest is in the destabilizing effect of the transverse force. This effect is greatest when the slip is zero, at which point a relatively simple expression for the normalized transverse force can be written as

$$F^*_t = 0.25 \left(\frac{1}{1 - \tilde{y}} - \frac{1}{1 + \tilde{y}}\right)\tag{7}$$

The transverse force–displacement characteristic is plotted in Fig. 5 (solid line). This characteristic indicates the instability of the motor. A small displacement toward one stator will cause a force on the slider in the same direction. The behavior is like that of a nonlinear, negative spring.

2.2. Example design

The purpose of this section is to show that, in principle, a useful motor design is feasible. The example we have chosen is the design of an “artificial muscle fiber,” which is a linear electrostatic motor exhibiting performance comparable to a single muscle cell. For present purposes, it will be assumed that some mechanism keeps the slider stabilized; it will be shown in Section 4 that fluid bearings provide such a mechanism while also having a minimal effect on performance.

\*The term “force” will be used in place of “force per unit length and width.”
The mechanical performance of a muscle fiber can be summarized in terms of a stress ($\sigma$, force per unit cross-sectional area) and a strain rate ($\gamma$, velocity per unit length). These quantities are independent of series and parallel concatenation, and therefore apply equally to bulk muscle. For vertebrate skeletal muscle, approximate maximum values are

$$\sigma \approx 3 \times 10^5 \text{ N/m}^2$$

$$\gamma \approx 8 \text{ s}^{-1}.$$  

Consider now the design of an electrostatic induction motor with comparable performance. The maximum motive force occurs at zero velocity, and equals $0.2\varepsilon_r (V/d)^2$, according to Eq. (5). The stress will be this value times the ratio of slider length ($l$) to motor width ($\sim 4d$); thus

$$0.2\varepsilon_r (V/d)^2 \frac{l}{4d} = 3 \times 10^5 \text{ N/m}^2. \quad (8)$$

If a silicone oil is chosen as a working fluid, then $\varepsilon_r \approx 2.6\varepsilon_0$, and a field strength of $V/d = 1.5 \times 10^7 \text{ V/m}$ will remain below breakdown strength. These values lead to an aspect ratio for the gap of $l/d = 1160$.

The strain rate specification provides another design constraint:

$$\frac{c}{l} = 8 \text{ s}^{-1}. \quad (9)$$

By arbitrarily selecting $d = 50 \mu m$, the aspect ratio and Eq. (9) can be used to compute the motor's length and peak speed: $l = 5.8 \text{ cm}$, $c = 46 \text{ cm/sec}$. The field strength constraint and gap width can be used to find the peak voltage: $V = 750 \text{ V}$. High voltages and low currents are characteristic of film-type electrostatic motors.  

It is important also to reassess the validity of the simplifying assumptions made in Section 2.1. For instance, the constraint $kd = 1$ requires that the stator potential have a wavelength of $314 \mu m$. In a three-phase arrangement, this would require 2 mil ($\sim 50 \mu m$) electrode widths and spaces, which is within the reach of current printed circuit technology. The excitation frequency can be determined from the wavelength and speed: $\omega = 236 \text{ Hz}$. The constraint $kt = 1$ requires that the slider be about $50 \mu m$ thick. Finally, the constraint $\sigma \omega = 1$ requires that the slider resistivity be $\sigma \approx 10^{-9}$ ($\Omega \text{ cm}$)$^{-1}$. This conductivity is far greater than that of silicone fluid, as desired, but it is large enough to be in the semiconductor range, and therefore pose a fabrication difficulty. Studies of amorphous silicon coatings for polyimide film sliders are now underway.

3. APPROACHES TO GAP MAINTENANCE

Film-type electrostatic motors pose the challenge of maintaining narrow, uniform gaps over large areas and under the influence of destabilizing electrostatic forces. This section reviews various approaches to “gap maintenance.”

Rolling element bearings are the standard means of gap maintenance in conventional electromagnetic motors. In the case of rotary motors, bearings prohibit radial motion, but provide little resistance to angular rotation. There are several problems, however, with the application of rolling element bearings to artificial muscle. Due to the narrowness and number of gaps, tolerance stack-up is a particularly severe problem. A sensible way to avoid this problem is to miniaturize the bearings, and build them into individual linear motors. This, however, leads to a host of difficulties, including bearing assembly, extremely tight tolerances, and rollers creating electric field concentrations, possibly leading to breakdown. Moreover, it is not even clear that rollers of $50 \mu m$ diameter will roll rather than slide.

In lieu of rolling element bearings, much simpler sliding bearings have been widely used in silicon motors. 21 Sliding bearings, however, tend not to be effective over the larger surface area of film motors. Moreover, sliding has clear disadvantages: friction reduces efficiency and expedites wear, and sliding promotes contact electrification which can cause motor failure. 21

A variety of other approaches have also been explored. For instance, Pister et al. 27 demonstrated a two-degree-of-freedom micromotor levitated by a forced-air bearing. Kim et al. 13 demonstrated a linear micromotor that used the Meissner effect to achieve levitation. Pelrine 23 has argued that materials, such as bismuth and graphite, exhibiting room temperature diamagnetism should make effective bearings for micromotors.

Electric forces themselves may be used to implement bearings. For instance, Kumar and Cho have used radio-frequency excitation of a tuned circuit including the electrostatic motor’s capacitance to demonstrate levitation. 14, 15 In Ref. 32, a film motor is described which achieves levitation based on an unusual scheme of four-phase slider excitation and three-phase stator excitation. An analytical study reported in Ref. 32 verifies that such an approach can produce levitation, but generally with a significant loss in motive force.

In the next section, a novel approach to gap maintenance based on fluid bearings is presented and investigated.

4. FLUID FILM GAP STABILIZATION

4.1. One-degree-of-freedom analysis

In this section, we consider the 2-D motor model shown in Fig. 4. For simplicity, we assume that the slider cannot tilt; i.e. the slider is free to translate along either the x- or y-axis, but cannot rotate about any axis. This assumption will be removed in Section 4.3. Here, the point is to demonstrate that fluid forces may be used to stabilize the transverse motion of the slider (i.e. maintain a gap), while having only a modest effect on the motor’s efficiency.

4.1.1. Model. The model is similar to that used in Section 2, except that the slider no longer has uniform
thickness. A “step” of height \( \delta \) on each side of the slider gives rise to stabilizing fluid forces, as will be demonstrated. It is assumed that the step has no effect on the expressions for motive and transverse force computed in Section 2; however, the nominal gap width is set to an average value: \( d = d_i - \zeta_o \delta \).

The fluid flow is assumed to be governed by the Navier–Stokes equation and appropriate boundary conditions. In addition, the following assumptions are made:

- Fluid inertia can be neglected. The Reynolds number, \( \rho\delta v/\mu \), is typically much less than 1.
- The fluid is incompressible and Newtonian.
- Variations in fluid properties due to temperature and pressure changes are neglected.
- The fluid films both above and below the slider are sufficiently thin that pressure gradients in the y-direction can be neglected.
- The y-direction velocity of the slider is approximately zero at all times \( y_r \approx 0 \). In the very narrow gaps of electrostatic motors, squeeze film effects dominate the dynamics, keeping transverse velocities very low. Assuming \( y_r = 0 \) focuses the analysis on displacement-modulated forces which are those ultimately responsible for transverse stability.
- Body forces do not exist (this stems from the assumption that the fluid does not sustain free charge).

4.1.2. Fluid forces. The fluid forces acting on the slider can be divided into transverse (y-directed) and drag (x-directed) components. Under the above set of assumptions, the fluid pressure will be governed by the Reynolds equation, and fluid forces easily determined. Those of interest are

\[
P_f = -\frac{d^2}{d y_r^2} \left( \frac{\mu v}{\zeta_o} \right) \left[ \frac{c_o}{d_o} + \frac{1 - \zeta_o}{d_i} - \frac{3 \zeta_o (1 - \zeta_o) \delta^2}{\zeta_o d_i^3 + (1 - \zeta_o) d_o^3} \right] \tag{10}
\]

\[
P_f = \frac{d^2}{d y_r^2} \left( \frac{\mu v}{\zeta_o V^2} \right) \left( \frac{1}{d} \right) \left[ \frac{3 \zeta_o (1 - \zeta_o) \delta}{\zeta_o (d_i - y_r)^2 (1 - \zeta_o) (d_o - y_r)^3} \right. \\
\left. - \frac{3 \zeta_o (1 - \zeta_o) \delta}{\zeta_o (d_i - y_r)^2 (1 - \zeta_o) (d_o - y_r)^3} \right] \tag{11}
\]

The normalized transverse force is plotted in Fig. 5 (dashed line). Unlike the electrostatic fields, the fluid acts to generate a restoring force upon transverse displacement of the slider. This may be understood from within the frame of the slider as follows. Fluid flowing underneath the slider (for instance) is squeezed by the step, which, due to the fluid’s viscosity, results in a positive pressure distribution that peaks at the location of the step. The same effect occurs above the slider, and the resulting forces balance when \( y_r = 0 \). This effect, however, depends on gap width, becoming more pronounced as the gap narrows and viscous forces increase. Thus, if the slider moves upwards \( (y_r > 0) \), a greater pressure will push it downwards than will push it upwards. Note that this effect occurs in one direction of motion only; thus, this model, like biological muscles, will pull but not push.

4.1.3. Transverse stability. The slider will be stable at \( y_r = 0 \) if the negative slope of the \( P_f(y_r) \) characteristic is greater than the positive slope of the \( \lambda_f \) characteristic:

\[
\frac{d P_f}{d y_r} \bigg|_{y_r=0} > \frac{d \lambda_f}{d y_r} \bigg|_{y_r=0} \tag{12}
\]

or

\[
\lambda_f \left( \frac{d}{d} \right) \left( \frac{\mu v}{\zeta_o V^2} \right) \left( \frac{d^2}{d y_r^2} \right) > 1 \tag{13}
\]

where

\[
\lambda_f = 18 \zeta_o (1 - \zeta_o) \delta \frac{\zeta_o d_i^2 + (1 - \zeta_o) d_o^2}{(\zeta_o d_i^3 + (1 - \zeta_o) d_o^3)^2}.
\]

Subsequent analyses are simplified by some further parameter selection. It has been argued in Ref. 17 that a reasonable choice of geometric parameters is \( \zeta_o = 0.625 \) and \( d_i/d_o = 1.2 \). With these parameters, \( \lambda_f = 0.64 \).

It is convenient to define a pair of nondimensional “levitation numbers”:

\[
N_f^* = \frac{\mu v}{\zeta_o V^2} \tag{14a}
\]
\[ N_{L}^{c} = \frac{\mu cl}{\varepsilon_f V^2}. \]  
\hspace{1cm} (14b)

The transverse stability condition can be written compactly in terms of either:
\[ N_{L}^{u} > 1.56 \]  
\hspace{1cm} (15a)
\[ v/c > 1.56/N_{L}^{c}. \]  
\hspace{1cm} (15b)

As is evident in Eq. (15b), there is always a minimum speed below which levitation will not occur.

4.1.4. Efficiency. If we assume that the slider is moving fast enough to levitate, then \( y_r = 0 \), and we may compute the motor's efficiency as a function of levitation number and speed. The output power is simply the net \( x \)-direction force times the speed. Using normalized quantities, this is
\[ P_{\text{out}} = \frac{V}{c} \left[ \sum F_x^e + \sum F_x^f \right]. \]  
\hspace{1cm} (16)

The input power is the time-averaged product of voltage and current,* which works out to be
\[ P_{\text{in}} = \sum F_x^e. \]  
\hspace{1cm} (17)

Thus, the efficiency can be written as
\[ \eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{V}{c} \left[ 1 + \frac{\sum F_x^f}{\sum F_x^e} \right] \]  
\hspace{1cm} (18)

or
\[ \eta = (1 - S) \left[ 1 - 2.5 N_{L}^{c} \left( \frac{d}{l} \right) \left( \frac{1 - S}{S} \left( 1 + S^2 \right) \right) \right] \]  
\hspace{1cm} (19)

where we have used the fact that, for the geometric parameters chosen:
\[ F_x^f (y_r = 0) = -\frac{d}{l} N_{L}^{c}. \]  
\hspace{1cm} (20)

Figures 6 and 7 show plots of \( \eta \) vs \( v/c \) and of \( \sum F_x^e + \sum F_x^f \) vs \( v/c \) for selected values of \( N_{L}^{c} \) and \( l/d \). The aspect ratio of the gap is evidently an important parameter. It is only for rather high aspect ratios that both high efficiency and a low critical velocity for levitation are possible.

4.1.5. Summary. Efficient operation of an electrostatic motor is, in principle, possible using fluid bearings for gap maintenance. The minimum speed required for stability is governed by the levitation number. High efficiency is favored by long, narrow gaps.

4.2. Design example (continued)
This is a continuation of the design example presented in Section 2.2. The objective now is to select an appropriate fluid viscosity. This may be done, for instance, by selecting \( N_{L}^{c} = 10 \) so that the minimum speed for levitation is approximately 26% of \( c \), then solving for \( \mu \):
\[ \mu = N_{L}^{c} \varepsilon_f \frac{V^2}{cl} = N_{L}^{c} \varepsilon_f \left( \frac{c}{l} \right) \left( \frac{l}{d} \right)^{-2} \left( \frac{V}{d} \right)^2 \]  
\hspace{1cm} (21)

\[ = 4.8 \times 10^{-3} \text{ N} - \text{s/m}^2. \]

This viscosity is about 5 times that of water, well within the range of commercially available silicone oils. This motor reaches a peak efficiency of 74% at a speed \( v/c = 0.86 \).

4.3. Multi-degree-of-freedom analysis
In Section 4.1, we considered a slider which was artificially constrained to translations along the \( x \)- and \( y \)-axes. Any real slider would, of course, have finite depth, and would not be so constrained. If rigid, as assumed throughout this paper, the slider would have six degrees-of-freedom, which we may identify with the

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*This assumes that power stored in the motor's capacitance can be removed and used at a later time; i.e. via inductance in the drive circuit. Many laboratory implementations simply use high-voltage transistors to switch electrode voltages on or off, and exhibit significantly reduced efficiency.
simplify analysis, we will assume that the slider is conductive and grounded, and the stators are conductive and held at the voltage $V/\sqrt{2}$ (the RMS voltage of an ideal induction motor). It is then evident that electrostatic forces would not influence rotation about the $y$-axis, or translation along the $z$-axis. Thus, the three degrees-of-freedom which are of interest are translation along the $y$-axis, and rotation about the $x$- and $z$-axes (roll and pitch, respectively).

The question to be addressed in this section is whether stability about these three degrees-of-freedom can be attained under conditions comparable to those found in the previous section, permitting high-efficiency operation. Should this not be the case, for instance, if tilting about one of the axes is considerably more difficult to stabilize, we would have to conclude that fluid levitation is not a viable approach to gap maintenance.

4.3.1. Approach. The method used to assess stability will be a generalization of that used in Section 4.1. First, the electrostatic forces and moments acting on the slider will be normalized and arranged into a vector, $[F_y^e \, M_y^e \, M_z^e]^T$. This vector will be differentiated with respect to the normalized configuration vector, $[\dot{y}, \, \dot{\theta}, \, \dot{\phi}]^T$. The resulting Jacobian matrix will be evaluated at the nominal configuration, $[\dot{y}, \, \dot{\theta}, \, \dot{\phi}] = [0 \, 0 \, 0]$, and the negative of this result will be termed the "electrostatic stiffness matrix," $K^e$. It will have the following form:

$$K^e = 
\begin{bmatrix}
k_{yy}^e & k_{y\theta}^e & k_{y\phi}^e \\
k_{\theta y}^e & k_{\theta\theta}^e & k_{\theta\phi}^e \\
0 & 0 & k_{\phi\phi}^e
\end{bmatrix}.
$$

Zero entries indicate that, about the nominal configuration, a small amount of roll will not affect either $F_y^e$ or $M_y^e$; likewise $M_z^e$ is unaffected by both pitch and displacements along the $y$-axis. The eigenvalues of $K^e$ ("eigennstiffnesses") are all negative, indicating that, taken alone, the electrostatic system is mechanically unstable.

A "fluid stiffness matrix," $K^f$, can be found by an exactly analogous procedure, and shows the same block structure as $K^e$:

$$K^f = N_k^f
\begin{bmatrix}
k_{yy}^f & k_{y\theta}^f & k_{y\phi}^f \\
k_{\theta y}^f & k_{\theta\theta}^f & k_{\theta\phi}^f \\
0 & 0 & k_{\phi\phi}^f
\end{bmatrix} = N_k^f \hat{K}^f.$$

The dependence on levitation number should be noted. Computational experience suggests that the eigenstiffnesses of $K^f$ are generally all positive at the nominal configuration, indicating that the fluid system taken alone is mechanically stable.

The eigenvalues of the "total stiffness matrix," $K = K^e + K^f$, are nonlinear functions of $N_k^f$ (actually, the eigenstiffness corresponding to roll stability is a linear function). For very small levitation numbers, the
eigenvalues approach those of \( K^e \), and, for very large levitation numbers, the eigenvalues approach those of \( K^f \); thus, there is guaranteed to be a minimum value of \( N^e_L \) for which all eigenstiffnesses are positive. The goal of this section may be restated as that of finding the minimum levitation number and comparing it to that found in Section 4.1 for the two-degree-of-freedom slider. For a given slider design, this problem is readily solved numerically.

4.3.2. Electrostatic stiffness. In addition to those mentioned above, major assumptions underlying the computation of electrostatic forces and moments are:

- Electric field lines between the slider and stator are everywhere parallel to the \( y \)-axis. Thus, fringing fields and distortions due to the step and slider tilting are ignored.
- If \( d_l(x, z) \) is the length of a field line impinging on the lower side of the slider at the location \( (x, z) \), the associated stress is

\[
\sigma_l(x, z) = -\frac{\varepsilon \varepsilon_0}{2d_l^2(x, z)} \left( \frac{V}{\sqrt{2}} \right)^2
\]  

(24)

or, normalized by the Maxwell stress and in terms of normalized coordinates \((\tilde{\xi} = x/l, \tilde{\eta} = z/w)\):

\[
\tilde{\sigma}_l(\tilde{\xi}, \tilde{\eta}) = -\frac{1}{4d_l^2(\tilde{\xi}, \tilde{\eta})}.
\]  

(25)

An analogous assumption applies to the upper side of the slider.

These assumptions reflect the facts that typical aspect ratios are very high (\( l/d \gg 1 \), \( w/d \gg 1 \)), and tilt angles are very small. Using small angle approximations, the functions \( \tilde{d}_l(\tilde{\xi}, \tilde{\eta}) \) and \( \tilde{d}_u(\tilde{\xi}, \tilde{\eta}) \) may be written terms of \( \tilde{y}, \tilde{\mu}, \tilde{\nu}, \tilde{\phi} \):

\[
\tilde{d}_l(\tilde{\xi}, \tilde{\eta}) = \tilde{d}_l + \tilde{\xi} \tilde{\theta} - \tilde{\xi} \tilde{\theta} + \tilde{\eta}, \quad \text{for} \ \tilde{\xi} \leq \zeta_o
\]

\[
\tilde{d}_l(\tilde{\xi}, \tilde{\eta}) = \tilde{d}_l + \tilde{\xi} \tilde{\theta} - \tilde{\xi} \tilde{\theta} + \tilde{\eta}, \quad \text{for} \ \tilde{\xi} > \zeta_o
\]

(26)

\[
\tilde{d}_u(\tilde{\xi}, \tilde{\eta}) = \tilde{d}_u - \tilde{\xi} \tilde{\theta} + \tilde{\xi} \tilde{\theta} - \tilde{\eta}, \quad \text{for} \ \tilde{\xi} \leq \zeta_o
\]

\[
\tilde{d}_u(\tilde{\xi}, \tilde{\eta}) = \tilde{d}_u - \tilde{\xi} \tilde{\theta} + \tilde{\xi} \tilde{\theta} - \tilde{\eta}, \quad \text{for} \ \tilde{\xi} > \zeta_o
\]

(27)

The electrostatic stiffness is computed a column at a time by solving for the force–moment vector in each of three configurations: \([\tilde{y}, \tilde{\mu}, \tilde{\nu}, \tilde{\phi}] = [\varepsilon \ 0 \ 0 \ 0], [0 \ \varepsilon \ 0 \ 0], \text{and} \ [0 \ 0 \ \varepsilon \ 0] \), where \( \varepsilon \ll 1 \). A numerical limit is taken to ensure the accuracy of the result. The force and moments are found by numerical integration of the following expressions:

\[
\tilde{F}_y = \int_{-1/2}^{1/2} \int_0^1 (\tilde{\sigma}_l(\tilde{\xi}, \tilde{\eta}) - \tilde{\sigma}_u(\tilde{\xi}, \tilde{\eta})) \, d\tilde{\xi} \, d\tilde{\eta}
\]

\[
\tilde{M}_\phi = \int_{-1/2}^{1/2} \int_0^1 \tilde{\phi}_l(\tilde{\xi}, \tilde{\eta}) - \tilde{\phi}_u(\tilde{\xi}, \tilde{\eta})) \, d\tilde{\xi} \, d\tilde{\eta}
\]

(28)

4.3.3. Fluid stiffness. In addition to those detailed in Section 4.1, an important assumption is:

- Pressure gradients in the \( z \)-direction are negligible; in other words, all flow exits the rear edge of the slider. In practice, flow will exist the sides of the slider. The error introduced by ignoring this effect becomes negligible only when the width of the slider is several times the length.

With this assumption, the pressure profiles above and below the slider can be found analytically. At any fixed value of \( z \), the slider may be treated as 2-D, having the step-shape shown in Fig. 4, but being capable of in-plane tilting. The solution for the pressure profile is given in the Appendix.

The fluid forces and moments acting on the slider are found by numerical integration of

\[
\tilde{F}_y = \int_{-1/2}^{1/2} \int_0^1 (\bar{p}_l(\tilde{\xi}, \tilde{\eta}) - \bar{p}_u(\tilde{\xi}, \tilde{\eta})) \, d\tilde{\xi} \, d\tilde{\eta}
\]

\[
\tilde{M}_\phi = \int_{-1/2}^{1/2} \int_0^1 \tilde{\phi}_l(\tilde{\xi}, \tilde{\eta}) - \tilde{\phi}_u(\tilde{\xi}, \tilde{\eta})) \, d\tilde{\xi} \, d\tilde{\eta}
\]

(29)

where \( \bar{p}_l(\tilde{\xi}, \tilde{\eta}) \) and \( \bar{p}_u(\tilde{\xi}, \tilde{\eta}) \) are the normalized pressures above and below the slider, given in the Appendix. The fluid stiffness is computed in the same fashion as the electrostatic stiffness, a column at a time. It is interesting to note that the fluid stiffness matrix is not symmetric (see below); thus, there is no associated potential energy function.

4.3.4. Stability. The normalized electric stress and fluid pressure depend on precisely the same parameters as in the 2-D case, \( \zeta_o, d_l/d_u \), and \( N^e_L \). The first two parameters, therefore, are given the same values as in Section 4.1, while the levitation number is left variable. The electrostatic and fluid stiffness matrices are found to be

\[
K^e = \begin{bmatrix}
-1.0475 & -0.4613 & 0 \\
-0.4613 & -0.2815 & -0.0873 \\
0 & 0 & 0
\end{bmatrix}
\]

\[
K^f = N^e_L \begin{bmatrix}
0.6402 & 0.6612 & 0 \\
0.3468 & 0.4494 & 0 \\
0 & 0 & 0.0534
\end{bmatrix}
\]

(30)

The total stiffness experienced by the slider is simply \( K = K^e + K^f \). The eigenstiffnesses of \( K \) are functions of \( N^e_L \), and are plotted in Fig. 9. As predicted, there is a minimum value of levitation number for which all three eigenstiffnesses are positive. In this case, the minimum is \( N^e_L = 2.654 \), which is 1.7 times greater than found for the 2-D model.
5. EXPERIMENTS

An experimental study was performed to verify the stability relation implied by the levitation number. The study involved the use of a specially-constructed apparatus rather than a linear motor.

The experimental apparatus is essentially a miniature tow tank. Two aluminum plates, lapped to meet a 5 μm flatness specification and anodized to provide insulation, serve as the tank walls (stators). The gap between the plates is maintained with precision spacers, then sealed with a rubber gasket along three edges and filled with silicone fluid. A stainless steel slider, suspended from a length of 37 gage magnet wire via a 32 gage stainless steel wire harness, is lowered into this gap. The harness allows the slider to rotate and translate freely in the potentially unstable degrees-of-freedom. The harness and magnet wire are also used to ground the slider electrically. The upper end of the magnet wire is fixed to one end of a balance beam, and is pulled upwards by adding weights to the other end of the beam. The angle of the beam is measured with an RVDT placed at its pivot. From this, the vertical position and velocity of the slider can be computed.

The stability of the slider is assessed in two ways. If stable, the slider should center itself between the stators (leaving a gap on either side of ~100 μm). In this location, the drag on the slider should be minimized (thus, the velocity should be maximized) and the capacitance between the slider and stators should also be minimized. Thus, both velocity and a measure of this capacitance were used as indicators of stability. Preliminary experiments established the repeatability of the capacitance measurement and verified the expected behavior in the absence of electric fields. These are detailed in Ref. 33.

Typical experimental results for stable and unstable bearings are shown in Fig. 10. When stable, the capacitance asymptotically approaches a minimum and the velocity a maximum. When unstable, the velocity drops to zero and the capacitance returns to a high value.

In a given set of experiments, the weight placed on the balance beam is fixed while the voltage is increased until instability is evident. Results are shown on a log scale, along with the theoretical prediction, in Fig. 11. The data are well fitted by a straight line and the slope is in close agreement with the theoretical prediction. The data show a significant offset, however, which can be attributed to the 2-D nature of the flow. Recall that the analysis assumed that no flow exited the sides of the slider where in fact flow must if the pressure at the edges is approximately gage, while the pressure under the step is higher. An analytical solution for the 2-D flow beneath a step-shaped slider (assuming no tilting) indicates that, for the geometry used in our experiment, side leakage will reduce load-bearing capacity by a factor of 2.5. This factor is very close to the factor of 2.3 predicted from the offset seen in Fig. 11. Although this effect appears significant, it would be greatly reduced in practice by making the z-direction width of the stator cavity only slightly greater than the width of the slider. The effect of side-leakage may also be

\[ y = -7.804 + 1.9775x, \quad R^2 = 0.99338 \]

Fig. 10. Plots of velocity and capacitance vs time for stable (dashed) and unstable (solid) conditions. The arrow indicates the time at which voltage is applied. In the stable case, the capacitance reaches and maintains a minimum value, indicating that the slider is centered between the two stators, even in the presence of an electric field. The velocity, moreover, reaches a maximum value which is unaffected by the field. In the unstable case, both capacitance and velocity are affected. The capacitance returns to a high value after application of the field, indicating that the slider approaches one stator. As it does so, the drag on the slider increases and the velocity decreases.

Fig. 11. Plot of critical velocity vs voltage. Experimental and theoretical results show nearly the same slope, but are significantly offset. Reasons for the offset are discussed in the text.
reduced by increasing the width-to-length ratio of the slider.

6. CONCLUSIONS

By properly exploiting unique features of electrostatic systems, such as the absence of return paths and the generation of fields by sheets of charge rather than regions of dipoles, aggregate motors can offer a realistic alternative to conventional magnetic motors, especially in the arena of motion control. One prerequisite to the success of a (film-type) electrostatic motor, however, is a high-efficiency gap maintenance system. In this paper, a novel approach to gap maintenance based on dielectric fluid bearings has been investigated. A simple model has been used to identify two important nondimensional parameters: a levitation number and a gap aspect ratio. An effective, efficient bearing results when both are maximized.

The bearing model developed here has, of course, been oversimplified in various respects. For instance, it has not considered stator or slider flexibility, or the more insidious problem of stator and slider curvature resulting from residual stresses. In early versions of film actuators, both problems have been minimized by mounting films in rigid frames; however, this may not be a suitable long-term solution.

A second obvious difficulty is start-up—the fluid bearing is not effective until some minimum speed is reached. This may be addressed by incorporating small stand-offs into the slider or stator to ensure the maintenance of some minimum gap. While stand-offs would increase friction at low velocities, they would not have a significant impact on efficiency, which is greatest at higher speeds. Further, in an artificial muscle containing many small linear motors connected end to end, low aggregate speeds could be achieved by activating only a very few individual motors, each of which would contract at a high enough speed for levitation.

A final point is that gap maintenance is only one of the challenges presented by electrostatic motors. Others include materials development (especially for induction motors, as considered here), the design of efficient voltage controllers, and miniaturization.

REFERENCES

APPENDIX

The pressure profiles above and below the slider may be found by integration of the Reynolds equation with proper boundary conditions (zero gage pressure at \( x = 0 \) and \( x = l \), and with an expression for the gap thickness as a function of location.\(^1\) For the gap beneath the slider, the normalized result is:

\[
\hat{p}(\hat{x}, \hat{z}) = -6\hat{N}_\ell \frac{\hat{x}}{d_1(d_1 + \hat{d}\hat{x})} \left[ 1 - \frac{2d_1 + \hat{d}\hat{x}}{d_1(d_1 + \hat{d}\hat{x})} \right]
\]  

(A1)

for \( \zeta_o \leq \hat{x} \leq 1 \):

\[
\hat{p}(\hat{x}, \hat{z}) = -6\hat{N}_\ell \frac{\hat{x} - 1}{d_1(d_1 + \hat{d} \hat{x})} \left[ 1 - \frac{2d_1 + \hat{d} \hat{x}}{d_1(d_1 + \hat{d} \hat{x})} \right]
\]

\[
\left\{ 1 - \frac{2d_2 + \hat{d}(1 + 2\zeta_o + \hat{x})}{d_2(1 + \zeta_o \hat{d})^3(d_2 + (\hat{x} + \zeta_o \hat{d})^2)} \right\}
\]

(A2)

where

\[
d_1 = \hat{d}_1 + \hat{y}, \hat{z}
\]

(A3)

\[
d_2 = \hat{d}_1 + \hat{y}, \hat{z}
\]

(A4)

\[
\hat{d}_n = \frac{1 - \zeta_o}{(d_2 + (1 + \zeta_o \hat{d})^2(d_2 + 2\zeta_0 \hat{d})^2)}
\]

\[
\left\{ (d_2 + (1 + \zeta_o \hat{d})^2(d_2 + 2\zeta_0 \hat{d})^2)^{-1} \right\}
\]

(A5)

\( \hat{d}_1 \) is the normalized gap thickness at the trailing edge of the slider; \( \hat{d}_1 + \zeta_0 \hat{d} \) is the normalized gap thickness at the location of the step; \( \hat{d}_n \) takes on the value necessary to ensure that pressure is continuous at the step. A similar result applies to the gap above the slider.