Passivity of a Class of Sampled-Data Systems: Application to Haptic Interfaces

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Received October 17, 1995; revised August 14, 1996; accepted August 27, 1996

Passivity of systems comprising a continuous time plant and discrete time controller is considered. This topic is motivated by stability considerations arising in the control of robots and force-reflecting human interfaces (''haptic interfaces''). Necessary conditions for passivity are found via a small gain condition, and sufficient conditions are found via an application of Parseval's theorem and a sequence of frequency domain manipulations. An example—implementation of a ''virtual wall'' via a one degree-of-freedom haptic interface—is given and discussed in some detail. *© 1997 John Wiley & Sons, Inc.*

この発表では、連続時間製造装置と不連続時間コントローラを含むシステムの、受動性について考察する。この話題は、ロボットと力感応型ヒューマン・インターフェイス（hapticインターフェイス）の制御における安定性の問題から来ている。受動性に対する必要条件は、低利得の条件下で見つけられ、十分条件はParseval理論の応用と、一連の周波数領域マニピュレータによって見つけられる。1 d.o.f.のhapticインターフェイスによる「仮想壁面」の実装例を紹介し、その詳細について考察する。

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Journal of Robotic Systems 14(1), 37–47 (1997)
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CCC 0741-2223/97/010037-11

1. INTRODUCTION

Passivity is a powerful tool for the analysis of coupled stability problems arising in robotics and related disciplines. For instance, passivity methods have been used to establish conditions for the stability of a robot contacting an uncertain dynamic environment,[1] to investigate the robustness of force feedback controllers,[2,3] and to study the stability of telemanipulation with a time delay.[4] Passivity is frequently used as a design constraint or objective in the development of manipulator controllers.[5–8] More recently, passivity techniques have been used in the design of "haptic interfaces" to virtual environments.[9] A haptic interface is a device that lets human operators touch, feel, and manipulate virtual (computer-generated) environments.[10–13]

Haptic interface design provides direct motivation for the problem considered in this article—passivity of sampled-data systems. This is because a haptic interface endowed with nearly ideal, collocated sensors and actuators, and implementing a virtual environment whose physical counterpart is passive, may nonetheless exhibit unstable oscillations when grasped by a human operator. Often, the frequency of these oscillations is outside the range of human voluntary movement and involuntary tremor, which would indicate that the energy required to sustain them is supplied by the interface. Thus, the interface is active, not passive. This is a direct consequence of the time delay and loss of information inherent in sampling.

Active behavior in robots often stems principally from noncollocation of actuators and sensors rather than sampling (thus, the works cited in the first paragraph focus exclusively on continuous time systems). Sampling, however, can be an important consideration if the sample rate is very low or if, as in direct drive robots, efforts are made to collocate actuators and sensors. Thus, the theory presented here should be applicable to certain problems that arise in robotics and telemanipulation, as well as haptic interfaces.

Prior work on passivity has focused on continuous-time and discrete-time systems, but has not addressed sampled-data systems (continuous-time plant and discrete-time controller).[14,15] Recently, however, there has been a growing interest in the derivation of *norms* for sampled-data systems. This interest is motivated principally by the application of such norms to $\mathbf{H}_2$ and $\mathbf{H}_\infty$ optimal controller design.[16–18] Conditions for passivity can, however, be converted via a bilinear transformation to conditions for the boundedness of an $\mathbf{L}_2$ induced norm. A formula for the $\mathbf{L}_2$ induced norm has been presented by Leung, et al.,[17] although this formula assumes a bandlimited input (to eliminate aliasing), an assumption that may not be valid for robotic and haptic interface applications in which collisions create high frequencies. Kabamba and Hara[19] present conditions for the boundedness of the $\mathbf{L}_2$ norm and a proposition concerning computation of the norm. Sivashankar and Khargonekar[18] present formulas for $\mathbf{L}_1$ and $\mathbf{L}_\infty$ induced norms, and an upper bound for the $\mathbf{L}_2$ induced norm.

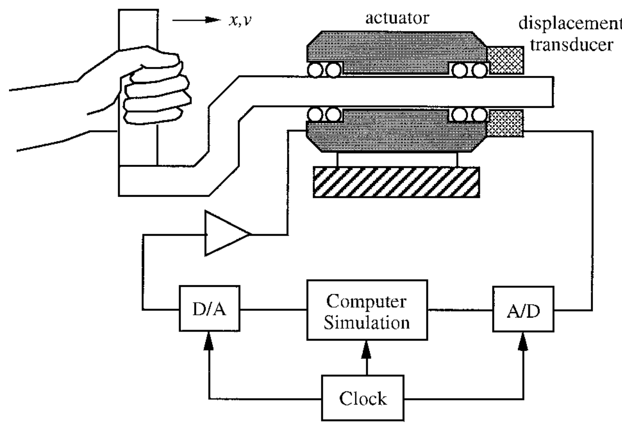

Figure 1. Schematic of a 1-DOF haptic interface.

2. PROBLEM STATEMENT

The problem will be described in terms of a prototypical 1 degree-of-freedom (DOF) haptic interface, picture in Figure 1. The interface basically consists of an actuator, such as a servomotor, which the operator grasps. Feedback signals representing the state of the interface are input to a computer. In this article, it is assumed that there is only one feedback signal, based on the displacement of the actuator. Although no such restriction need be observed in practice, this is a common implementation. The computer calculates an actuator command according to its model of the virtual environment. This command is output, amplified, and sent to the actuator.

Figure 2 is a model of this system. It is assumed that the actuator and handle behave as a rigid body (m) with some viscous friction (b), acted upon by a controller force (u). Amplifier and sensor dynamics, nonlinearity, and noise are ignored. The feedback signal is sampled at the rate T, and the control signal is passed through a zero order hold. The virtual environment (feedback controller) is represented by a stable linear, shift-invariant transfer function, $H(z)$.

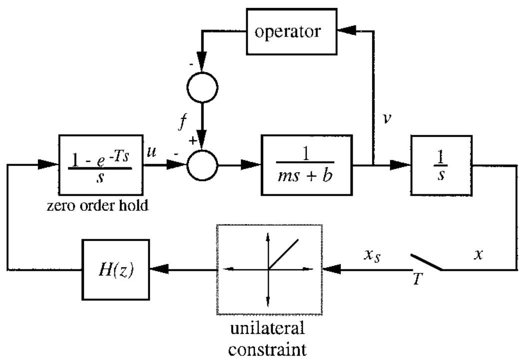

Figure 2. Model of a 1-DOF haptic interface.

Useful virtual environments cannot be composed strictly of linear operators, however. At a minimum, it is necessary to include the nonlinear element pictured in Figure 2. This element, the unilateral constraint, is ubiquitous in the physical world. An example is the constraint experienced by a ball dropped on a floor—vastly different equations of motion apply when the ball is and is not in contact with the floor. Unilateral constraints are needed, in general, to account for collisions and contact. They are, however, nonlinear. In this article, necessary and sufficient conditions for passivity will first be found for the linear case. It will then be shown that the sufficient conditions apply equally if a unilateral constraint is incorporated as in Figure 2.

If the haptic display behaves passively, then the operator can never extract energy from it. Here, we will use the slightly more stringent statement that the energy input to the haptic display from the operator must be positive for all admissible force histories $f(t)$ (see discussion in section 4) and all times greater than zero:

$$\int_0^t f(\tau)\,v(\tau)\,d\tau > 0, \forall t > 0, \text{admissible} f(t) \qquad (1)$$

A system that does not satisfy Eq. (1) is said to be "active." Section 3 develops a necessary and sufficient condition for Eq. (1). Section 4 presents an example and discusses in some detail the implications for haptic interface.

3. AN ANALYTICAL PASSIVITY CRITERION

The major result of this article is given by the following theorem:

Theorem 1: *A necessary and sufficient condition for passivity of the sampled data system in Figure 2 (without the unilateral constraint) is:*

$$b > \frac{T}{2}\frac{1}{1-\cos\omega T}\text{Re}\{1-e^{-j\omega T})H(e^{j\omega T})\} \qquad 0 \le \omega \le \omega_N \qquad (2)$$

where $\omega_N = \pi/T$ is the Nyquist frequency.

3.1. Proof of Necessity

One of the well-known consequences of passivity is the following: a strictly passive system, connected to any passive environment, is necessarily stable. Thus, stability when connected to a linear time-invariant, passive, but otherwise arbitrary environment may be considered a necessary condition for passivity (interestingly, for linear time-invariant systems this is also a sufficient condition[1]). This idea is the basis of the necessity proof.

Suppose that the unilateral constraint in Figure 2 is removed and the operator is replaced with a passive, but otherwise arbitrary impedance $Z_O(s)$. The closed loop characteristic equation of the resulting system is:

$$1 + H(e^{sT})G^*(s) = 0 \qquad (3)$$

where:

$$G^*(s) = \frac{1}{T}\sum_{n=-\infty}^{n=\infty} G(s + jn\omega_s) \qquad (4)$$

$$G(s) = \frac{1-e^{-Ts}}{s^2}\frac{1}{ms + b + Z_O(s)} \qquad (5)$$

and $\omega_s = 2\pi/T$. It will be proved that Eq. (2) is necessary to ensure that Eq. (3) contains no unstable roots. The proof requires the use of coupled stability theory.[1,14,20–22] For clarity, the approach is first outlined: To begin, the constraint that $Z_O(s)$ is passive is used to identify the region of the Nyquist plane, $\mathbf{R}_{G^*}$, within which $G^*(j\omega)$ must lie at each frequency. Next, a linear fractional transformation, $\mathbf{M}\{j\omega, G^*(j\omega)\}$ (defined below,* is found which will map this region to the *complete* interior of the unit disk, pointwise in frequency. The results of Colgate[23] are then used to find a related

*A linear fractional transformation (LFT) is a type of conformal mapping having the property that it maps circular regions in the complex plane to other circular regions (this includes half planes, which are considered circles of infinite extent).

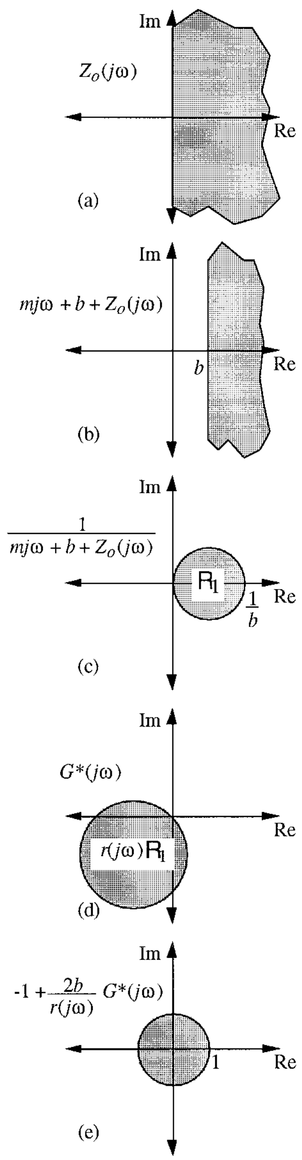

Figure 3. Sequence of Nyquist plane transformations.

linear fractional transformation (LFT), $\mathbf{N}\{s, H(e^{sT})\}$, through which $H(e^{sT})$ can be mapped such that the closed loop characteristic equation, written in terms of transformed quantities, contains the same unstable poles as Eq. (3). This ensures that the transformation does not alter closed loop stability. The Small Gain Theorem then leads directly to a necessary and sufficient condition for closed loop stability: $\|\mathbf{N}\{s, H(e^{sT})\}\|_\infty < 1$.

$\mathbf{R}_{G^*}$ is found via a series of transformations on the region corresponding to $Z_O(j\omega)$, as illustrated in Figure 3. The latter is simply the closed right half plane[14]: $\mathrm{Re}\{Z_O(j\omega)\} \geq 0$, $\mathrm{Im}\{Z_O(j\omega)\}$ arbitrary. Consider first the term $mj\omega + b + Z_O(j\omega)$. The purely imaginary contribution of the mass will have no effect on the region, while the damping will shift the entire region to the right by b units. The resulting half plane is shown in Figure 3(b). Next consider the term $(mj\omega + b + Z_O(j\omega))^{-1}$. The half plane is mapped, via the inverse, to a closed disk centered on the real axis at $1/2b$ with a radius of $1/2b$. Note that this is true at every frequency. Let this region be denoted $\mathbf{R}_1$.

Because $\mathbf{R}_1$ is frequency-independent, it can be removed from the infinite sum when computing the region corresponding to $G^*(j\omega)$:

$$\mathbf{R}_{G^*}(\omega) = r(j\omega)\mathbf{R}_1 \tag{6}$$

where:

$$r(j\omega) = \frac{1}{T}\sum_{n=-\infty}^{\infty} \frac{1 - e^{-(j\omega + jn\omega_s)T}}{(j\omega + jn\omega_s)^2} \tag{7a}$$

$$= \frac{1 - e^{-j\omega T}}{T}\sum_{n=-\infty}^{\infty} \frac{1}{(j\omega + jn\omega_s)^2} \tag{7b}$$

$$= \frac{T}{2}\frac{e^{-j\omega T} - 1}{1 - \cos\omega T} \tag{7c}$$

The factor $r(j\omega)$ may be viewed as a frequency-dependent rotation and scaling. Thus, $\mathbf{R}_{G^*}(\omega)$ is a disk at each frequency (Fig. 3(d)), and it may be mapped to the unit disk by a translation and scaling. An appropriate LFT, applied to $G^*(s)$, is:

$$\mathbf{M}\{s, G^*(s)\} = -1 + \frac{2b}{r(s)} G^*(s) \tag{8}$$

Thus, the region corresponding to $\mathbf{M}\{j\omega, G^*(j\omega)\}$ is the closed unit disk centered at the origin (Fig. 3(e)). Details for finding the associated transformation $\mathbf{N}\{s, H(e^{sT})\}$, are given in ref 23. The result is:

$$\mathbf{N}\{s, H(e^{sT})\} = \frac{r(s)H(e^{sT})}{2b + r(s)H(e^{sT})} \tag{9}$$

It can be verified by direct computation that the closed loop characteristic equation of the transformed system has the same unstable roots as those of the original system:

$$1 + \mathbf{M}\{s, G^*(s)\}\mathbf{N}\{s, H(e^{sT})\}$$

$$= 1 + \left[-1 + \frac{2b}{r(s)}G^*(s)\right]\left[\frac{r(s)H(e^{sT})}{2b + r(s)H(e^{sT})}\right] \quad (10a)$$

$$= \frac{1 + H(e^{sT})G^*(s)}{1 + \frac{r(s)}{2b}H(e^{sT})} \quad (10b)$$

The poles of $H(e^{sT})$ are also roots, but are assumed stable; $r(s)$ introduces imaginary poles at integer multiples of the sampling frequency. Because $\mathbf{M}\{j\omega, G^*(j\omega)\}$ is completely uncertain in phase and may have a magnitude as great as one, the Small Gain Theorem[14] gives a necessary and sufficient condition for stability, which is $\|\mathbf{N}\{s, H(e^{sT})\}\|_\infty < 1$, or:

$$\left|\frac{r(j\omega)H(e^{j\omega T})}{2b + r(j\omega)H(e^{j\omega T})}\right| < 1 \ \forall\omega \quad (11)$$

Straightforward manipulation then leads to the alternate expression in Eq. (2). The periodicity of $r(j\omega)H(e^{j\omega T})$ has also been used to narrow the range of frequencies that must be examined to $0 \le \omega \le \omega_N$.

3.2. Proof of Sufficiency

Consider again the system in Figure 2, and suppose the mass is initially at rest. An intuitive statement of passivity is that the kinetic energy of the mass never be as great as the total energy input by the source $f(t)$:

$$\frac{1}{2}mv^2(t) < \int_0^t f(\tau)v(\tau)\,d\tau, \forall t > 0, \text{admissible } f(t) \quad (12)$$

Because kinetic energy is positive definite, satisfaction of Eq. (12) is clearly a sufficient condition for passivity according to Eq. (1). Also, if kinetic energy ever exceeded total energy input, energy could be extracted simply by applying a force pulse sufficient to bring the mass to rest in an arbitrarily short time period. We conclude, therefore, that Eq. (12) is equivalent to Eq. (1).

A force balance on the mass leads to the following:

$$\frac{1}{2}mv^2(t) = \int_0^t f(\tau)v(\tau)\,d\tau - \int_0^t u(\tau)v(\tau)\,d\tau - \int_0^t b\,v(\tau)v(\tau)\,d\tau \quad (13)$$

Subtracting Eq. (13) from Eq. (12) gives:

$$\int_0^t u(\tau)v(\tau)\,d\tau + \int_0^t b\,v^2(\tau)\,d\tau > 0, \quad \forall t > 0, \text{admissible } v(t), \dot{v}(t) \quad (14)$$

An admissible signal is one for which the truncated $\mathbf{L}_2$ norm:

$$\|v\|_t \triangleq \left[\int_0^t v^2(\tau)\,d\tau\right]^{1/2} \quad (15)$$

is non-zero and finite for all t. The restriction to admissible velocity and acceleration in Eq. (14) ensures that the same class of signals is covered as in Eq. (12). Given any admissible $f(t)$, the admissible signals $v(t)$ and $\dot{v}(t)$ can be found by direct integration; given any admissible $v(t)$, $\dot{v}(t)$, an admissible $f(t)$ can be found by solving the inverse dynamics.

The passivity condition (Eq. (14)) may be converted to a frequency domain condition using Parseval's Theorem. First, define a class of truncated signals:

$$v_\theta(\tau) = \begin{cases} 0 & \tau < 0 \\ v(\tau) & 0 \le \tau \le \theta \\ 0 & \tau > \theta \end{cases} \quad (16)$$

Equation (14) can be rewritten as:

$$\int_{-\infty}^{\infty} u(\tau)v_t(\tau)\,d\tau + \int_{-\infty}^{\infty} b\,v_t^2(\tau)\,d\tau > 0, \quad \forall t, \text{admissible } v(t), \dot{v}(t) \quad (17)$$

Parseval's Theorem gives an equivalent inequality:

$$\int_{-\infty}^{\infty} U(j\omega)V^*(j\omega)\,d\omega + \int_{-\infty}^{\infty} b\,V(j\omega)V^*(j\omega)\,d\omega > 0, \quad \forall\omega, \text{admissible } V(j\omega) \quad (18)$$

where $U(j\omega)$ and (admissible) $V(j\omega)$ are Fourier Transforms of $u(\tau)$ and $v_t(\tau)$, respectively.

The signal U can be written in terms of V using the sampler, pulse transfer function, and zero order hold. (Refer to Fig. 2). Using the impulse modulation model of a sampler, the spectrum of the sampled position signal may be written:

$$X^S(j\omega) = \frac{1}{T}\sum_{n=-\infty}^{\infty}\frac{V(j\omega + jn\omega_S)}{j\omega + jn\omega_S} \quad (19)$$

Thus:

$$U(j\omega) = \frac{1 - e^{-j\omega T}}{j\omega T} H(e^{j\omega T}) \sum_{n=-\infty}^{\infty} \frac{V(j\omega + jn\omega_S)}{j\omega + jn\omega_S} \quad (20)$$

The following definition is made to simplify notation:

$$\overline{H}(\omega) = -\frac{1 - e^{-j\omega T}}{T} H(e^{j\omega T}) \quad (21)$$

$\overline{H}(\omega)$ is periodic with a period equal to the sampling rate, T. Equation (18) may now be rewritten as:

$$\int_{-\infty}^{\infty} \overline{H}(\omega) \sum_{n=-\infty}^{\infty} \frac{V(j\omega + jn\omega_S)}{j\omega + jn\omega_S} \left[\frac{V(j\omega)}{j\omega}\right]^* d\omega + \int_{-\infty}^{\infty} b\, V(j\omega)V^*(j\omega)\, d\omega > 0 \quad (22)$$

It can be shown that the value of the first integral in Eq. (22) is unchanged if $\overline{H}(\omega)$ is replaced by $\mathrm{Re}\{\overline{H}(\omega)\}$. The requisite manipulations, which are somewhat tedious, are presented in Appendix A.1.

The next step is to identify an analytical lower bound to the sum of integrals in Eq. (22). To do this, we first define $\Phi^+(\omega)$ and $\Phi^-(\omega)$ as follows:

$$\Phi^+(\omega) = \begin{cases} \mathrm{Re}\{\overline{H}(\omega)\} & \forall\omega \mid \mathrm{Re}\{\overline{H}(\omega)\} \geq 0 \\ 0 & \forall\omega \mid \mathrm{Re}\{\overline{H}(\omega)\} < 0 \end{cases} \quad (23a)$$

$$\Phi^-(\omega) = \begin{cases} 0 & \forall\omega \mid \mathrm{Re}\{\overline{H}(\omega)\} \geq 0 \\ \mathrm{Re}\{\overline{H}(\omega)\} & \forall\omega \mid \mathrm{Re}\{\overline{H}(\omega)\} < 0 \end{cases} \quad (23b)$$

In terms of these functions, the leftmost integral in Eq. (22) may be rewritten as follows:

$$I_0 = \int_{-\infty}^{\infty} \mathrm{Re}\{\overline{H}(\omega)\} \sum_{n=-\infty}^{\infty} \frac{V(j\omega + jn\omega_S)}{j\omega + jn\omega_S} \left[\frac{V(j\omega)}{j\omega}\right]^* d\omega = I_1 + I_2 \quad (24)$$

where:

$$I_1 = \int_{-\infty}^{\infty} \Phi^+(\omega) \sum_{n=-\infty}^{\infty} \frac{V(j\omega + jn\omega_S)}{j\omega + jn\omega_S} \left[\frac{V(j\omega)}{j\omega}\right]^* d\omega \quad (25a)$$

$$I_2 = \int_{-\infty}^{\infty} \Phi^-(\omega) \sum_{n=-\infty}^{\infty} \frac{V(j\omega + jn\omega_S)}{j\omega + jn\omega_S} \left[\frac{V(j\omega)}{j\omega}\right]^* d\omega \quad (25b)$$

First, a lower bound is found for I_1. By defining $\zeta(\omega) = (\Phi^+(\omega))^{1/2}V(j\omega)/j\omega$, I_1 may be rewritten as follows, and shown to be positive for all admissible $V(j\omega)$:

$$I_1 = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \zeta(\omega + n\omega_S)\zeta(\omega)^*\, d\omega \quad (26a)$$

$$= \int_{-\infty}^{\infty} z(t) \sum_{n=-\infty}^{\infty} z(t)e^{-jn\omega_s t}\, dt$$

$$= T\int_{-\infty}^{\infty} z^2(t) \frac{1}{T} \sum_{n=-\infty}^{\infty} e^{-jn\omega_s t}\, dt \quad (26b)$$

$$= T\int_{-\infty}^{\infty} z^2(t) \sum_{k=-\infty}^{\infty} \delta(t - kT)\, dt \quad (26c)$$

$$= T\sum_{k=-\infty}^{\infty} z^2(t - kT)\, dt \quad (26d)$$

$$\geq 0 \quad (26e)$$

Here, the convolution property and the Fourier Transform of an ideal sampler have been used. Equation (26e) indicates that 0 is a lower bound for I_1. A lower bound for I_2 is derived in Appendix A.2. It is:

$$I_2 \geq \int_{-\infty}^{\infty} \Phi^-(\omega) \sum_{n=-\infty}^{\infty} \frac{1}{(\omega + n\omega_s)^2} V(j\omega)V^*(j\omega)\, d\omega \quad (27)$$

Inserting Eqs. (26e) and (27) into Eq. (22), a sufficient condition for passivity is:

$$\int_{-\infty}^{\infty} \left[b + \Phi^-(\omega) \sum_{n=-\infty}^{\infty} \frac{1}{(\omega + n\omega_s)^2}\right] V(j\omega)V^*(j\omega)\, d\omega > 0 \quad (28)$$

This inequality will be satisfied for all admissible $V(j\omega)$ if and only if the expression in brackets is positive at every frequency. This is because the power in $v(t)$ can be concentrated in an arbitrarily narrow frequency band. Using the same equality as in Eq. (7), the sufficient condition may be rewritten:

$$b + \Phi^-(\omega)\frac{T^2}{2}\frac{1}{1 - \cos\omega T} > 0\ \ \forall\omega \quad (29)$$

It is easily verified that an equivalent condition is:

$$b + \mathrm{Re}\{\overline{H}(\omega)\}\frac{T^2}{2}\frac{1}{1 - \cos\omega T} > 0\ \ \forall\omega \quad (30)$$

Finally, it can be seen that Eqs. (30) and (2) are equivalent. This completes the proof in the absence of a unilateral constraint.

3.3. Sufficiency with a Unilateral Constraint

Because the output of the sampled data controller is fixed during each period, one can always construct a function $v(t)$, $kT \leq t < (k+1)T$, which will extract an arbitrarily large amount of energy from the actuator. By selecting large enough b (according to Eq. (2)), however, one is assured that at least as much energy will be lost to friction. Thus, one implication of the sufficiency proof is that, while moving, the haptic interface will consume energy during each and every sample period. Because, in addition, the haptic interface is passive in the absence of a feedback loop, we may conclude that $u(t)$ can be set to zero during any sample period without affecting the sufficiency result. In other words, the kinetic energy of the mass will at no point be greater than the total energy input by $f(t)$. Thus, the sufficiency proof guarantees that the incorporation of a unilateral constraint will not affect the passivity result obtained with a given linear controller.

4. EXAMPLE

This section considers a common implementation of a ''virtual wall,'' composed of a virtual spring and damper in mechanical parallel, together with a unilateral constraint operator.[9] A velocity estimate is obtained via backward difference differentiation of position, giving the following transfer function within the wall:

$$H(z) = K + B\frac{z-1}{Tz} \tag{31}$$

where $K > 0$ is a virtual stiffness, and $B > 0$ is a virtual damping coefficient. From Eq. (2), the condition for passivity is:

$$b > \frac{T}{2}\frac{1}{1-\cos\omega T}\operatorname{Re}\left\{(1-e^{-j\omega T})\left(K + B\frac{e^{j\omega T}-1}{Te^{j\omega T}}\right)\right\}$$
$$0 \leq \omega \leq \omega_N \tag{32}$$

This relation can be reduced by straightforward algebraic manipulation to:

$$b > \frac{KT}{2} - B\cos\omega T \qquad 0 \leq \omega \leq \omega_N \tag{33}$$

The right-hand side is maximized at $\omega = \omega_N$, leading to the condition:

$$b > \frac{KT}{2} + B \tag{34}$$

This result shows that, to achieve passivity, some physical dissipation is essential. It also shows that, given fixed physical and virtual damping, the maximum achievable virtual stiffness is proportional to the sampling rate. Further, the achievable virtual damping is independent of the sampling rate.

While these are useful guidelines for design, it is important to recognize that passivity is a potentially conservative design requirement. This is because, even if an interface is active, a human operator may not be able to destabilize it. To understand this point more fully, consider the following problem: given a virtual wall implementation that marginally violates Eq. (34), what passive physical behavior must the operator take on to cause instability? In other words, if we think of the operator as the interface's environment, what is the most destabilizing environment?

According to Eq. (33), active behavior is first evidenced at the Nyquist frequency. The goal, therefore, is to find a specific realization of $Z_O(s)$ such that the Nyquist plot of $G^*(s)H(e^{sT})$ just touches -1 when $\omega = \omega_N$. At this frequency, H takes on a particularly simple form:

$$H(e^{j\pi}) = K + \frac{2B}{T} \tag{35}$$

Assuming marginal violation of the passivity criterion:

$$\frac{KT}{2} + B = b \tag{36}$$

Combining Eqs. (35) and (36), the expression for $H(e^{j\pi})$ can be rewritten:

$$H(e^{j\pi}) = \frac{2b}{T} \tag{37}$$

It is easily verified that, if $G(s)$ satisfies the following condition:

$$G(s)\big|_{s=j(1+2n)\omega_N} = \frac{1}{ms + b + Z_O(s)}\bigg|_{s=j(1+2n)\omega_N} = \frac{1}{b}$$
$$n = \ldots -2, -1, 0, 1, 2 \ldots \tag{38}$$

the condition for marginal stability is met:

$$G^*(j\omega_N) = r(j\omega_N)\frac{1}{b} = -\frac{T}{2b} \quad (39)$$

There are many possible choices for $Z_O(s)$ that will satisfy Eq. (38). Thus, there is no single most destabilizing environment. One class of solutions may be found as follows. Let:

$$ms + Z_O(s) = \frac{ms(s^2 + \omega_2^2)(s^2 + \omega_4^2) \cdot \cdot \cdot}{(s^2 + \omega_1^2)(s^2 + \omega_3^2) \cdot \cdot \cdot} \quad (40)$$

$$\omega_i = \begin{cases} \frac{i}{2}\omega_N & i \text{ even} \\ \omega_{i-1} \leq \omega_i \leq \omega_{i+1} & i \text{ odd} \end{cases}$$

Note that Eq. (40) is zero at all interger multiples of $j\omega_N$. It can be shown that $Z_O(s)$ found by subtracting ms from both sides of Eq. (40) is a positive real function.[24] $Z_O(s)$ may be viewed as the impedance of an infinite, lossless, but non-uniform transmission line.

Clearly, this is not a behavior that we can expect a human operator to emulate. On the other hand, by eliminating higher order ($i > 2$) terms in Eq. (40) and selecting $\omega_1 = 0$, we find that an approximation to the most destabilizing environment is:

$$Z_O(s) \approx \frac{ms(s^2 + \omega_N^2)}{s^2} - ms = \frac{m\omega_N^2}{s} \quad (41)$$

This is simply the impedance of a spring which, together with the mass of the interface, resonates at the Nyquist frequency. Certainly it is much more plausible that a human operator might emulate a spring.[25]* A question of interest, therefore, is the extent to which passivity is conservative compared to a condition based on stability when coupled to springs.

To compute the latter condition, termed ''spring stability,'' $Z_O(s)$ is replaced with k/s in Figure 2, $H(z)$ from Eq. (31) is introduced, and conditions under which the coupled system is stable for arbitrary k are found. This is achieved by a straightforward numerical procedure that solves for the roots of the z-domain closed loop characteristic equation. The results are presented in Figure 4 in terms of three nondimensional parameters:

$$\alpha = \frac{KT}{b} \qquad \beta = \frac{B}{b} \qquad \tau = \frac{m}{bT} \quad (42)$$

Also shown in Figure 4 are conditions for the stability of the interface in the absence of any $Z_O(s)$, termed ''uncoupled stability.'' The passivity condition depends on α and β only ($\alpha/2 + \beta < 1$), but the other two conditions depend on τ as well. τ is the ratio of the mechanical time constant of the haptic interface (m/b) to the sampling period.

Figure 4 shows that, for small enough $\tau(\tau = 0.1)$, the uncoupled stability and spring stability regions of the α-β plane converge on the passivity region. In this case, passivity is not a conservative condition. For higher values of τ, however, the three regions separate. For $\tau = 10$, passivity is substantially more conservative than spring stability, which is in turn substantially more conservative than uncoupled stability.

These results have certain implications for haptic interface design. To implement very stiff, dissipative constraints (high K, B), it is helpful to maximize b and minimize T. Fast sampling is a standard objective, but maximizing damping goes against conventional wisdom.[26] It is generally argued that the dynamics of a haptic interface should be dominated by the virtual environment (which is, after all, the programmed behavior we wish to display) rather than any inherent dynamics (which is considered parasitic). Unfortunately, this ignores the effect of sampling. Sampling ensures a certain disparity between the actual and intended behaviors of the virtual environment, which will result in active behavior and the potential for coupled instability unless accompanied by a sufficient degree of inherent damping (b). It is quite interesting to note, however, that the passivity condition does not rule out the use of *negative* virtual damping. For instance, if $B < 0$ is permitted, the passivity condition (Eq. (34)) changes to:

$$b > \frac{KT}{2} + |B| \quad (43)$$

Thus, as much negative virtual damping is permissible as positive virtual damping. In the case of $K = 0$, it should be possible to eliminate almost completely the effect of inherent damping.

Although increasing b decreases α and β, it also decreases τ, which reduces the usable range of α and β

*Even so, the maximum stiffness of the operator may be well below that predicted by Eq. (41), depending on m and ω_N.

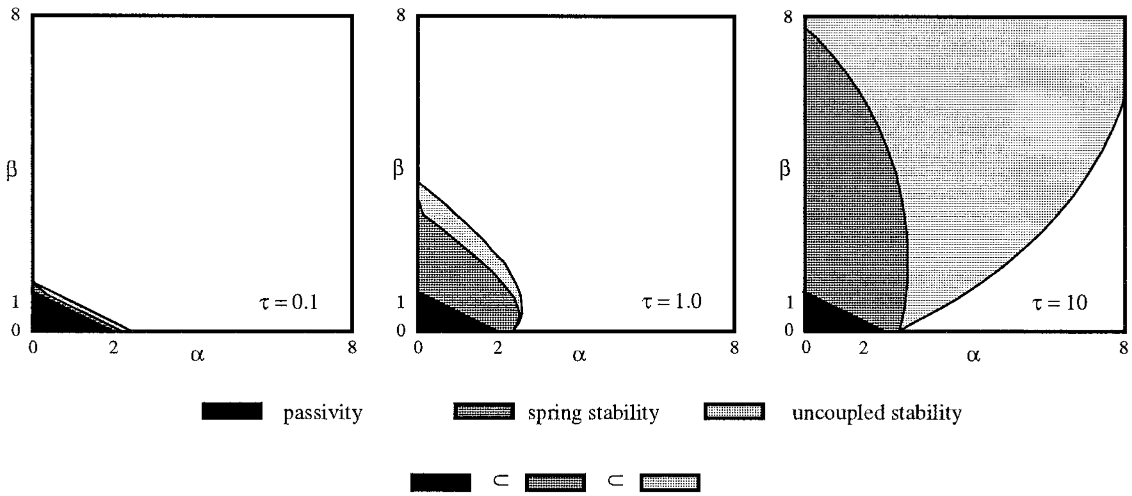

Figure 4. Regions of stability. Note that uncoupled stability includes spring stability, which includes passivity.

(assuming that this range is given by spring stability). This effect may be offset by increasing m, which also runs contrary to conventional wisdom. Whether inherent mass can be effectively eliminated via acceleration feedback is questionable. Acceleration signals, especially those obtained via differentiation, tend to be noisy; moreover, the human hand is quite sensitive to even very small amounts of mechanical vibration caused by propagated noise. Filtering is a possible solution, but the delay introduced by a filter tends to compromise passivity. Thus, in the proper mix of inherent and virtual dynamic behaviors lies a number of interesting problems for future research.

5. CONCLUSIONS

A necessary and sufficient condition for the passivity of a class of sampled-data systems has been derived. The example of a "virtual wall" characterized by virtual stiffness and damping coefficients has been given and investigated in detail with the aid of the passivity condition. It has been shown that the implementation of a stiff, passive virtual wall is aided by large physical damping and high sampling rate. In addition, high inertia helps to ensure that passivity is a conservative coupled stability criterion for realistic sets of operator behavior. In other words, high inertia is a desirable design feature for haptic interfaces from the standpoint of coupled stability. Of course, in such a case, passivity criteria are not terribly useful design tools.

An important area for future research, therefore, is the derivation of less conservative coupled stability criteria for sampled data systems. Such criteria are not likely to be analytical, so this problem is tantamount to the development of efficient numerical techniques and data presentation techniques. Methods that may be applied to multi-input, multi-output systems are of particular importance.

A related problem deserving careful investigation is the effect of quantization (or sensor resolution) on passivity. Quantization is as fundamental a consequence of digital control as sampling. The roundoff generated by quantization may be viewed as a form of high frequency noise. This noise may be amplified by differentiation, leading to sustained oscillations in a haptic interface.

Quantization is, of course, a nonlinear effect. In addition to quantization, we can expect a great variety of nonlinear effects to be encountered in haptic interface, especially as increasingly complex virtual environments are designed and implemented. Thus, an important direction for future research seems to be toward understanding the coupled stability properties of nonlinear sampled-data systems.

APPENDIX A.1

It is shown that:

$$I_0 = \int_{-\infty}^{\infty} \overline{H}(\omega) \sum_{n=-\infty}^{\infty} \frac{V(j\omega + jn\omega_S)}{j\omega + jn\omega_S} \left[\frac{V(j\omega)}{j\omega}\right]^* d\omega$$

$$= \int_{-\infty}^{\infty} \mathrm{Re}\{\overline{H}(\omega)\} \sum_{n=-\infty}^{\infty} \frac{V(j\omega + jn\omega_S}{j\omega + jn\omega_S} \left[\frac{V(j\omega)}{j\omega}\right]^* d\omega \quad \text{(A1)}$$

To begin, the original expression is broken into two parts. This involves interchanging the order of integration and summation:

$$I_0 = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{-n\omega_s/2} \overline{H}(\omega) \frac{V(j\omega + jn\omega^S)}{j\omega + jn\omega_S} \left[\frac{V(j\omega)}{j\omega}\right]^* d\omega$$
$$+ \sum_{n=-\infty}^{\infty} \int_{-n\omega_s/2}^{\infty} \overline{H}(\omega) \frac{V(j\omega + jn\omega_S)}{j\omega + jn\omega_S} \left[\frac{V(j\omega)}{j\omega}\right]^* d\omega \quad \text{(A2)}$$

The following change of variable is introduced in the first integral on the right hand side of (A2): $\omega_1 = -\omega - n\omega_S$:

$$I_0 = \sum_{n=-\infty}^{\infty} \int_{-n\omega_s/2}^{\infty} \overline{H}(-\omega_1) \frac{V(-j\omega_1)}{-j\omega_1} \left[\frac{V(-j\omega_1 - jn\omega_S)}{-j\omega_1 - jn\omega_S}\right]^* d\omega_1$$
$$+ \sum_{n=-\infty}^{\infty} \int_{-n\omega_s/2}^{\infty} \overline{H}(\omega) \frac{V(j\omega + jn\omega_S)}{j\omega + jn\omega_S} \left[\frac{V(j\omega)}{j\omega}\right]^* d\omega \quad \text{(A3)}$$
$$= \sum_{n=-\infty}^{\infty} \int_{-n\omega_s/2}^{\infty} (\overline{H}(-\omega)$$
$$+ \overline{H}(\omega)) \frac{V(j\omega + jn\omega_S)}{j\omega + jn\omega_S} \left[\frac{V(j\omega)}{j\omega}\right]^* d\omega \quad \text{(A4)}$$

It is easily verified that $\overline{H}(-\omega) = \overline{H}^*(\omega)$, so that (A4) may be rewritten:

$$I_0 = \sum_{n=-\infty}^{\infty} \int_{-n\omega_s/2}^{\infty} 2\,\mathrm{Re}\{\overline{H}(\omega)\}$$
$$\frac{V(j\omega + jn\omega_S)}{j\omega + jn\omega_S} \left[\frac{V(j\omega)}{j\omega}\right]^* d\omega \quad \text{(A5)}$$

As a final step, it can be verified that the integrand is symmetric about $\omega = -\frac{n\omega_s}{2}$. This leads directly to the result in (A1).

APPENDIX A.2.

It is shown that:

$$I_2 = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \Phi^-(\omega) \frac{V(j\omega + jn\omega_S)}{j\omega + jn\omega_S} \left[\frac{V(j\omega)}{j\omega}\right]^* d\omega$$
$$\geq \int_{-\infty}^{\infty} \Phi^-(\omega) \sum_{n=-\infty}^{\infty} \frac{1}{(\omega + n\omega_S)^2} V(j\omega) V^*(j\omega)\, d\omega \quad \text{(A6)}$$

To begin, the order of integration and summation is interchanged, and the summation is broken into three parts (corresponding to zero, positive and negative values of n):

$$I_2 = \int_{-\infty}^{\infty} \Phi^-(\omega) \frac{V(j\omega)}{j\omega} \left[\frac{V(j\omega)}{j\omega}\right]^* d\omega$$
$$+ \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} \Phi^-(\omega) \frac{V(j\omega + jn\omega_S)}{j\omega + jn\omega_S} \left[\frac{V(j\omega)}{j\omega}\right]^* d\omega$$
$$+ \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} \Phi^-(\omega) \frac{V(j\omega - jn\omega_S)}{j\omega - jn\omega_S} \left[\frac{V(j\omega)}{j\omega}\right]^* d\omega \quad \text{(A7)}$$

Introducing the changes of variables $\omega_1 = \omega - n\omega_S$ in the third integral leads to:

$$I_2 = \int_{-\infty}^{\infty} \Phi^-(\omega) \frac{V(j\omega)}{j\omega} \left[\frac{V(j\omega)}{j\omega}\right]^* d\omega$$
$$+ \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} \Phi^-(\omega) \frac{V(j\omega + jn\omega_S)}{j\omega + jn\omega_S} \left[\frac{V(j\omega)}{j\omega}\right]^* d\omega$$
$$+ \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} \Phi^-(\omega) \frac{V(j\omega_1)}{j\omega_1} \left[\frac{V(j\omega_1 + jn\omega_S)}{j\omega_1 + jn\omega_S}\right]^* d\omega_1 \quad \text{(A8)}$$
$$= \int_{-\infty}^{\infty} \Phi^-(\omega) \frac{V(j\omega)}{j\omega} \left[\frac{V(j\omega)}{j\omega}\right]^* d\omega$$
$$+ \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} \Phi^-(\omega) \left[\frac{V(j\omega + jn\omega_S)}{j\omega} \left[\frac{V(j\omega)}{j\omega}\right]^*\right.$$
$$\left. + \frac{V(j\omega)}{j\omega + jn\omega_S} \left[\frac{V(j\omega + jn\omega_S}{j\omega}\right]^*\right] d\omega \quad \text{(A9)}$$

Using the inequality:

$$[BC^* + CB^*] \leq [BB^* + CC^*] \quad \text{(A10)}$$

and the fact that $\Phi^-(\omega)$ is negative definite, a lower bound is found:

$$I_2 \geq \int_{-\infty}^{\infty} \Phi^-(\omega) \left|\frac{V(j\omega)}{j\omega}\right|^2 d\omega$$
$$+ \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} \Phi^-(\omega) \left[\left|\frac{V(j\omega + jn\omega_S}{j\omega}\right|^2\right.$$
$$\left. + \left|\frac{V(j\omega)}{j\omega + jn\omega_S}\right|^2\right] d\omega \quad \text{(A11)}$$

The second integral in (A11) can be broken into two parts, and in the first of these the change of variables $\omega_1 = \omega + n\omega_S$ is made. This leads to the following:

$$I_2 \geq \int_{-\infty}^{\infty} \Phi^-(\omega)\left\{\left|\frac{V(j\omega)}{j\omega}\right|^2 + \sum_{n=1}^{\infty}\left[\left|\frac{V(j\omega)}{j\omega - jn\omega_S}\right|^2 + \left|\frac{V(j\omega)}{j\omega + jn\omega_S}\right|^2\right]\right\} d\omega \qquad \text{(A12)}$$

which is equivalent to the right hand side of (A6).

The authors gratefully acknowledge the support of the National Science Foundation, grants MSS-9022513 and IRI-9213234, and valuable discussions with Mike Stanley and Paul Millman.

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