On the Role of Dissipation in Haptic Systems

Brian E. Miller, J. Edward Colgate, and Randy A. Freeman

Abstract—Passivity theory has been used for the past decade to derive stability conditions for human/machine interface applications. Demonstrating passivity of the haptic display implies stable and safe interaction for the human user. At the heart of the stability analysis is the physical dissipation provided by the haptic device, as it plays a key role in the design process for all components. This paper will derive the condition that the haptic device must satisfy in order to achieve passivity[1] of the haptic display. These results will be used to investigate a general nonlinear device model.

Index Terms—Haptics, passivity, stability.

I. INTRODUCTION

Haptic systems are sampled-data systems that consist of an electro-mechanical *haptic* device (essentially, a robot), human operator, and discrete-time virtual environment. Each component of the haptic system has been the topic of research in disciplines ranging from psychology to engineering. The fact that the human operator is in direct contact with the haptic device has required the ability to demonstrate stability of the overall haptic system.

Many device designs use direct current (DC) motors for the method of actuation. The PHANToM [6] uses brushed DC motors to achieve force feedback in the three translational directions, and provides a thimble as the interface to the user. Others have used novel actuators in their design. Examples include a magnetically levitated joystick [8] that conveys electromagnetic forces, and the *Rutgers Ankle* [3] that uses pneumatic actuation.

Several researchers have addressed the stability of haptic systems. Adams [1] created a framework to derive stability conditions for both admittance and impedance haptic systems. Hannaford [4] contributed an energy-based method that uses an online passivity measure which prompts a passivity controller to adjust an adaptive dissipation element. Stramigioli *et al.* [10] presented a discretization technique targeted at preserving the passivity characteristics of port-Hamiltonian systems, and demonstrated how to connect continuous- and discrete-time port-Hamiltonian systems in a passive manner.

In this paper, stability of the haptic system will be assessed through a passivity-based framework. From a passivity viewpoint, the important feature is the dissipation (damping) provided by the haptic device. The conditions derived in the subsequent sections result in sufficient conditions for passivity. The relationships that involve device damping can then be directly related to virtual environment design to determine valid parameter ranges.

[1]For haptic systems consisting of linear elements, we strive for passivity of the haptic display. If the haptic system exhibits nonlinear behavior, then we strive to achieve pseudopassivity, a level between cyclopassivity and passivity.

II. PRELIMINARIES

A. Passivity

A passive system is one for which the maximum amount of energy that can be extracted is equal to the initial stored energy plus the amount of energy input in the system. A compelling method to mathematically express passivity is in terms of storage functions, which can be used to characterize both linear and nonlinear systems. The first step in using this approach is to identify a valid storage function W that satisfies the inequality in *Definition 1*. Identifying a suitable storage function can sometimes be a difficult task. Fortunately, for electromechanical systems, the systems of interest in this paper, the stored energy often turns out to be a valid choice for the storage function.

The continuous-time version of passivity states the following.

Definition 1: A system with input u and output y is *passive* if a nonnegative function W (called a storage function) exists, that is a function of the states (x), such that

$$W(x(t)) \leq W(x(0)) + \int_0^t y(\tau)u(\tau)d\tau - \delta \int_0^t y^2(\tau)d\tau$$
$$-\varphi \int_0^t u^2(\tau)d\tau \quad \forall x \in R^n, x, t \geq 0 \quad (1)$$

with $\delta = \varphi = 0$.

Input strict passivity (ISP) is achieved when $\delta = 0$ and $\varphi > 0$, and output strict passivity (OSP) is achieved when $\delta > 0$ and $\varphi = 0$. The discrete-time representation is obtained by replacing the integrals with summations.

A mapping from input to output will be referred to as being δ-OSP or φ-ISP, indicating passivity levels of δ and φ, respectively. This characterization is equivalent to saying that a system dissipates energy *at a rate δ or φ* (e.g., a viscous damper with damping coefficient B dissipates energy *at a rate B*). A mapping is referred to as having a lack of ISP or OSP if passivity (or strict passivity) can be achieved through a parallel or feedback connection of a static gain (or an appropriately defined transfer function), respectively.

Once the individual components have been characterized in terms of energy, the overall stability of a system can be achieved by using coupled stability results with the help of loop transformations. A loop transformation [5] provides an equivalent, yet expanded, view of the coupled system by identifying the locations and rates of energy dissipation. The basic approach uses the concept behind the circle criterion [5].

III. PROBLEM FORMULATION

The haptic device under consideration is shown in Fig. 1. Here we see the human operator (H), haptic device (D), and virtual environment (E). In this particular configuration, the haptic device is referred to as an impedance device, because it commands a torque (derived from force $F(t)$, which is the zero-order hold (ZOH) of the discrete-time force signal F) and measures position (x). The direction of the arrows in the bottom half of Fig. 1 were purposely left unspecified, because either an environment (E) with admittance (accepts force and returns velocity) or impedance (accepts velocity and returns force) causality can be programmed with the proper use of a virtual coupling[2] (V) [2]. Signals F_h and v_h represent the force and velocity, respectively, at the point the haptic device contacts the human.

[2]The virtual coupling is typically implemented as a discrete-time spring/damper in mechanical parallel. It is significant in the stability result for haptic systems with admittance virtual environments.
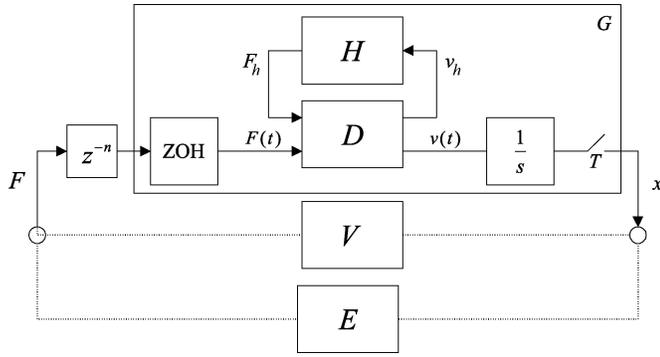
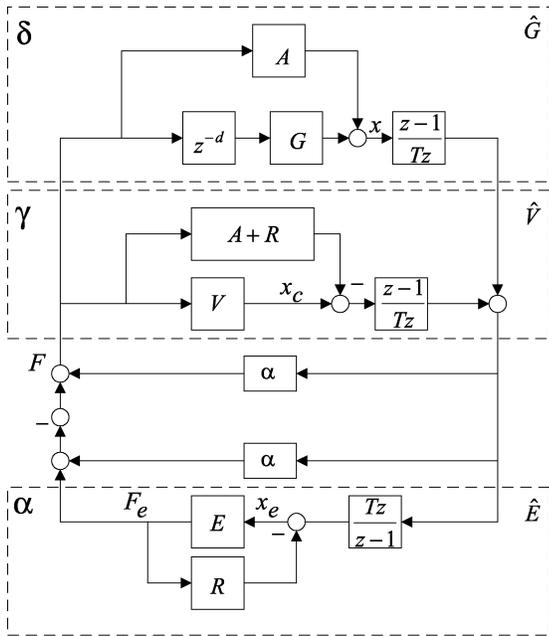Fig. 1.   Block diagram of the haptic system under consideration.

Fig. 2.   Loop-transformed haptic system.

A. Transformed Haptic System

The first step in analyzing the haptic system is to perform loop transformations to identify sources of dissipation. A loop-transformed version of the haptic system under consideration is shown in Fig. 2. It is necessary at this point to identify the desired virtual environment causality, because it impacts the type of loop transformations that must be performed. Virtual environments with impedance causality will be considered in this paper, however, a similar development exists for virtual environments with admittance causality.

Parameters δ and γ represent levels of strict passivity (dissipation) that the human/device and virtual coupling must satisfy, respectively. The loop transformation that introduced block A was performed to achieve a δ-OSP human/device block-once discretization, computational delay, and the sample/hold operator were included. This loop transformation adds a constraint on the passivity condition that the virtual coupling must satisfy. The loop transformations that introduced blocks R and α were performed to address energy growth that is common in virtual-environment models. For example, results in [7] demonstrate that unless a nonlinear virtual stiffness is nondecreasing, energy will be produced by the virtual environment.

The loop transformations define the transformed human/device block $\hat{G}$, transformed virtual coupling $\hat{V}$, and transformed virtual environment $\hat{E}$. Transformed blocks $\hat{G}$ and $\hat{V}$ must produce a strictly

passive combination, while $\hat{E}$ can exhibit a lack of passivity. The rate at which the environment can produce energy is determined by the dissipation rate of the device/virtual-coupling connection. In this transformed (but equivalent) representation, the circle criteria can be used to guarantee stability.

B. Transformed Human/Device

The previous section established that in order to guarantee stable interaction, the human/device feedback loop must satisfy strict passivity conditions. This section reveals that this requirement adds a design constraint to the haptic device. Basically, it implies that the haptic device must be capable of physically dissipating energy (satisfy continuous-time strict passivity) in order for the transformed human/device block to be discrete-time-output strictly passive.

For a (discrete-time) δ-OSP $\hat{G}$ to exist, it is assumed that the haptic device exhibits a damping characteristic of level δ. Let P denote the continuous-time human/device feedback loop in state-space form (Fig. 3)

$$P \begin{cases} \dot{w} = f(w, F) \\ v = h(w, F) \end{cases} \tag{2}$$

with internal state $w(t)$, input $F(t)$, and output $v(t)$. The vector field $f(\cdot, F)$ is assumed locally Lipschitz continuous and complete for each constant F, so we may define its parameterized flow $\phi(t, w_0, F)$, which denotes the state solution at time t for constant input F with initial condition w_0 at time zero.

To obtain a transformed human/device block $\hat{G}$ which is δ-OSP, we make the following choice for the block $A(z)$ in Fig. 2:

$$A(z) = \frac{T}{2\delta} \frac{(z^d + z^{d-1} + \ldots + z + 1)}{z^d}. \tag{3}$$

It is then clear that $\hat{G}$ defined in Fig. 2 is the same as the bottom part of Fig. 3. To write down a discrete-time state-space representation of $\hat{G}$, however, it is convenient to further transform the bottom part of Fig. 3 to the equivalent top part. Using the top part of Fig. 3, we obtain the following state-space representation of $\hat{G}$:

$$q_1(k+1) = u(k)$$
$$\vdots$$
$$q_{d+1}(k+1) = q_d(k)$$
$$q_{d+2}(k+1) = \phi\left(T, q_{d+2}(k), q_{d+1}(k)\right)$$
$$y(k) = \frac{1}{T} \int_0^T h\left(\phi\left(\tau, q_{d+2}(k), q_{d+1}(k)\right), q_{d+1}(k)\right) d\tau$$
$$+ \frac{1}{2\delta}\left[u(k) - q_{d+1}(k)\right]. \tag{4}$$

In the above definition, $q(k) = [q_1^T(k) \cdots q_{d+2}^T(k)]$ is the state, $u(k)$ is the discrete input, $y(k)$ denotes the output, $T > 0$ is the sample period, and $\delta > 0$ is a passivity parameter important in the following lemma.

Lemma 2: If P is (continuous-time) δ-OSP, then $\hat{G}$ is (discrete-time) δ-OSP.

Proof: Let $V(w)$ be a storage function for P, with device-damping characteristic δ. V is semidefinite and satisfies

$$V(w(t)) \le V(w_0) + \int_0^t F^T(\tau) v(\tau) d\tau - \delta \int_0^t v^T(\tau) v(\tau) d\tau. \tag{5}$$

We propose the following storage function for $\hat{G}$:

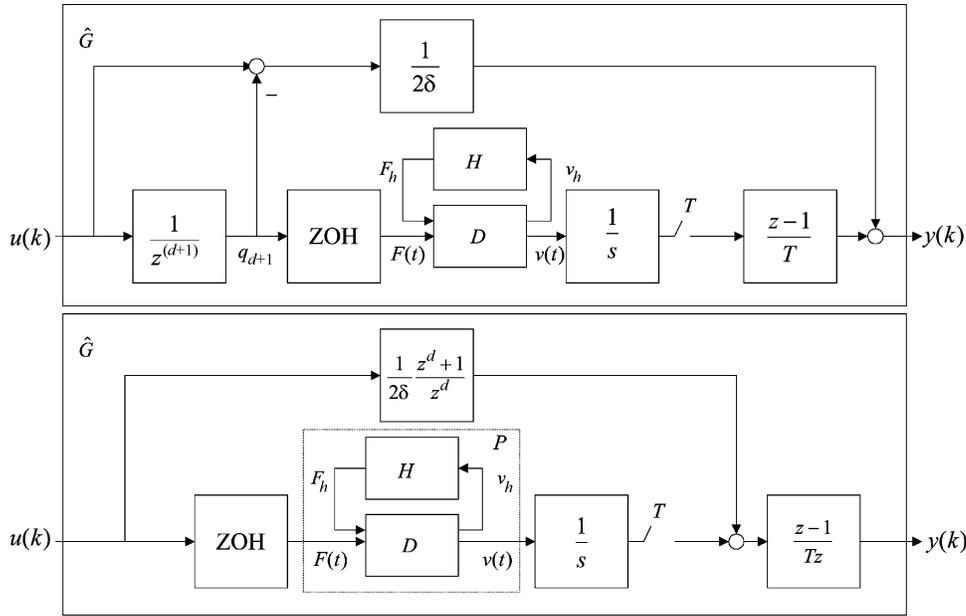$$W(q) = \frac{1}{4\delta}\left[q_{d+1}^T q_{d+1} + \cdots + q_1^T q_1\right] + \frac{1}{T} V(q_{d+2}). \tag{6}$$

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Fig. 3. Details of the transformed human/device subsystem.

To prove the lemma, we will demonstrate

$$W\left(q(k+1)\right) \leq W\left(q(k)\right) + y^T(k)u(k) - \delta y^T(k)y(k). \quad (7)$$

For constant input $F(t) \equiv q_{d+1}$ at time instant $t = T$ with initial condition $w_0 = q_{d+2}$, we can rewrite (5) as follows:

$$V\left(q_{d+2}(k+1)\right)$$
$$\leq V\left(q_{d+2}(k)\right) + q_{d+1}^T \int_0^T v(\tau)d\tau - \delta \int_0^T v^T(\tau)v(\tau)d\tau$$
$$\leq V\left(q_{d+2}(k)\right) + q_{d+1}^T \left[x(k+1) - x(k)\right] - \delta \int_0^T v^T(\tau)v(\tau)d\tau. \quad (8)$$

Applying the Cauchy–Schwarz inequality to the last term

$$\int_0^T v^T(\tau)v(\tau)d\tau \geq \frac{1}{T}\left|\int_0^T v(\tau)d\tau\right|^2 = \frac{1}{T}\left|x(k+1) - x(k)\right|^2. \quad (9)$$

We now have

$$V\left(q_{d+2}(k+1)\right) \leq V\left(q_{d+2}(k)\right) + q_{d+1}^T\left[x(k+1) - x(k)\right] - \frac{\delta}{T}\left|x(k+1) - x(k)\right|^2. \quad (10)$$

From (4), we observe that

$$x(k+1) - x(k) = T\left[y(k) - \frac{1}{2\delta}u(k) + \frac{1}{2\delta}q_{d+1}(k)\right] \quad (11)$$

thus

$$\frac{\delta}{T}\left|x(k+1) - x(k)\right|^2$$
$$= T\left[\delta y^T(k)y(k) + \frac{1}{4\delta}u^T(k)u(k) + \frac{1}{4\delta}q_{d+1}^T(k)q_{d+1}(k) - u^T(k)y(k) + q_{d+1}^T y(k) - \frac{1}{2\delta}q_{d+1}^T(k)u(k)\right]. \quad (12)$$

Plugging (11) and (12) into (10), and dividing through by T, we obtain

$$\frac{1}{T}V\left(q_{d+2}(k+1)\right) \leq \frac{1}{T}V\left(q_{d+2}(k)\right) + y^T(k)u(k)$$
$$- \delta y^T(k)y(k) - \frac{1}{4\delta}\left[u^T(k)u(k) - q_{d+1}^T(k)q_{d+1}(k)\right]. \quad (13)$$

Taking note of the chain of delays in (4)

$$q_{d+1}^T(k)q_{d+1}(k) - u^T(k)u(k)$$
$$= \left(q_d^T(k+1)q_d(k+1) - q_d^T(k)q_d(k)\right)$$
$$+ \cdots + \left(q_1^T(k+1)q_1(k+1) - q_1^T(k)q_1(k)\right) \quad (14)$$

therefore, (13) can be used to achieve (7). ∎

The above development relies on device damping to prove the existence of an (discrete-time) output strictly passive mapping from $u(k)$ to $y(k)$. The mathematical assumption that needs to be verified is that the continuous-time mapping from end-point force F to end-point velocity v is output strictly passive. It should be noted that the mapping from F to v was represented as P in *Lemma 2*.

The remainder of this section will focus on device model D. In so doing, it is necessary to consider the details contained in block D. The diagrams presented thus far have shown D as providing a mapping from end-point force F to end-point velocity v. In practical implementation, the end-point forces are transformed into joint torques, and then sent to a digital amplifier that determines the amount of current sent to the motor. Positions of the motors are sampled, and the end-point position is computed using the forward kinematics. Fig. 4 shows the details of this process. End-point forces (F) and velocities (v) are mapped into joint torques (τ) and velocities $(\dot{q})$, respectively, using the manipulator Jacobian (J)

$$\tau = J^T F \quad (15)$$
$$v = J\dot{q}. \quad (16)$$

The model of the haptic device that accepts $F(t)$ and returns $v(t)$ assumes that it is valid to model the forward kinematics and the transpose of the Jacobian (J^T) as continuous-time operations within the device model (D). The forward kinematics are static mappings, and therefore can be modeled on either side of the sample operation. The mapping of the end-point force (F) to joint torque (τ) through the transpose of the manipulator Jacobian commutes with the ZOH, when the Jacobian appears constant from the perspective of the virtual coupling and virtual
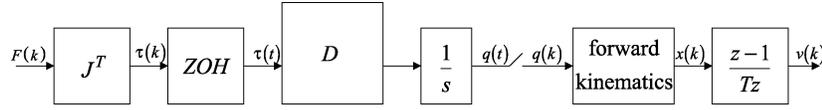
Fig. 4.   Details of the mapping from discrete-time end-point force $F(k)$ to discrete-time end-point velocity $v(k)$.

environment. This can be accomplished by sampling the haptic device at a faster rate than the rest of the haptic system. This topic is beyond the scope of this paper, however, we will assume that it has been satisfied. Under this assumption, it is valid to include J^T as part of the device model.

The final section will demonstrate how to determine the amount of inherent damping in a haptic device.

IV. INHERENT DEVICE DAMPING

A. General Nonlinear Device Model

The dynamics of a manipulator (haptic device), in joint space, can be represented using Langrangian dynamics

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + \delta_m \dot{q} = \tau. \tag{17}$$

Joint position and velocity are represented by q and $\dot{q}$, respectively, with τ representing motor torque. Variable δ_m represents a damping parameter in joint space. For clarity, the development that follows considers the single-degree-of-freedom (DOF) case with δ_m as a scalar. However, the multi-DOF analysis is a natural extension, where δ_m is a matrix of dissipation parameters.

Spong [9] demonstrated that in the absence of viscous friction (δ_m), the mapping $\dot{q} \to \tau$ is passive. The following will demonstrate that including viscous friction makes the mapping $\dot{q} \to \tau$ output strictly passive. Then the analysis will be transformed into end-effector space to achieve the OSP mapping from end-point force F to end-point velocity v.

We begin in joint space, where the mapping $\tau \to \dot{q}$ is OSP if there exists a δ_m, $E_0 > 0$ such that

$$\int_0^T \dot{q}^T \tau(\zeta)d\zeta \geq - \int_0^T \dot{q}^T \delta_m \dot{q} - E_0 \tag{18}$$

where E_0 represents the initial stored energy. The sum of the kinetic (M) and potential energy (U) is proposed as a candidate storage function

$$E = \frac{1}{2}\dot{q}^T M(q)\dot{q} + U(q). \tag{19}$$

Computing the change in energy $\dot{E}$ and performing a substitution using (17)

$$\dot{E} = \frac{1}{2}\dot{q}^T \dot{M}(q)\dot{q} + \dot{q}^T \left[M(q)\ddot{q} + G(q)\right] \tag{20}$$

$$= \frac{1}{2}\dot{q}^T \dot{M}(q)\dot{q} + \dot{q}^T \left[\tau - C(q,\dot{q})\dot{q} - \delta_m \dot{q}\right]. \tag{21}$$

The acceleration terms cancel, and using the skew-symmetry property $(\dot{M}(q) - 2C(q,\dot{q}) = 0)$, we are left with

$$\dot{E} = \dot{q}^T \tau - \dot{q}^T \delta_m \dot{q}. \tag{22}$$

Integrating both sides of (22), we recover (18), and the mapping $\tau \to \dot{q}$ is OSP.

We relate this expression to end-point force F and end-point velocity v using (15) and (16)[3]

$$\dot{E} = v^T F - v^T (J^T)^{\dagger} \delta_m J^{\dagger} v. \tag{23}$$

[3] $J^{\dagger} = J^T(JJ^T)^{-1}$ represents the pseudoinverse of the device Jacobian.

Thus, device dissipation in joint space (δ_m) relates to dissipation in end-effector space (δ) the following way:

$$\delta = (J^T)^{\dagger} \delta_m J^{\dagger}. \tag{24}$$

Viewing (24) from a device-design standpoint, motivates the following theorem.

Theorem 3: If a certain level of damping (δ) is needed to display a particular class of virtual environments, then the damping in the joints must satisfy

$$\delta_m \geq J^T \delta J. \tag{25}$$

Since the Jacobian is a function of motor position and velocity, practical use of (25) requires knowledge of workspace limitations and velocity limits. One interesting observation is that singularities within the workspace (Jacobian becomes singular) makes (25) easier to satisfy.

V. CONCLUSION

This paper has derived sufficient conditions for the passivity of the haptic display that involves device damping. Once the damping characteristic is known for a particular haptic device, the design of the virtual coupling and virtual environment can be pursued.

Classes of virtual environments can be parameterized and related directly to the damping characteristic of the device. Once the allowable class of virtual environments has been identified, the designer can decide if it is acceptable. If a larger class is desired, then the analysis can be used to determine how much to increase the device damping to allow the desired virtual environment design.

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