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Effusive flow conductance of shielded circular channels

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A circular shield coaxial with the conductance channel and spaced a small distance from the channel entrance is used to establish a well defined gas density gradient inside the effusion cell. The gradient and the resultant channel conductance are calculated by previously presented methods with the additional approximation that the plane of entrance to the channel is taken to be a partial absorber and a partial diffuse reflector with the absorption coefficient being the conventional Clausing factor of the channel. Transmission coefficients are calculated for a number of reduced geometries, and a practical means of fabricating shielded conductance elements is described.

Introduction

The free molecular conductance of cylindrical channels has been studied by Clausing[1,2] and others[3–7]. The familiar Clausing factors are commonly used to compute the rate of gaseous effusion through channels connecting regions of low pressure. The transmission characteristics of circular orifices and of short circular channels has been reinvestigated in order to include an effect which was not considered in earlier works. The latter were based on the assumption that the gas upstream of the entrance to the orifice or channel is in Maxwellian equilibrium and that departures from equilibrium set in abruptly as the flow crosses that plane. It is desirable to soften that rigid restraint by including the partial encroachment of the vaccum backwards into the gas chamber. The result will always be a reduction in the net flow, even for the case of a circular aperture of radius a and of negligible channel length for which the conductance is generally taken to be[4] $\bar{v}\pi a^2/4$ where $\bar{v}$ is the mean molecular speed.

Because the steady state characteristics of any molecular flow are determined by the geometric arrangement of sources, sinks and reflective walls, it is inadequate to tabulate a single transmission coefficient such as a Clausing factor for a single conductance element. The actual transmission coefficient will depend not only on the dimensions of the channel but also on the geometry of the rest of the cell. The presence of other conductance elements, for example, can have a pronounced effect on the net flow because of the increased density gradients they cause inside the cell. Because the number of different cell geometries which may be of practical importance is so great, it is not presently appropriate to attempt an attack on the problem sufficientiy broad to cover all or most of them. Instead we consider here a case in which the cell geometry in the immediate vicinity of the conductance element is arranged in some standard, easily producible way such that the effective radiative density gradients over all of the surfaces exposed to the channel can be determined and the resultant transmission coefficient calculated. The arrangement selected is shown schematically in Figure 1. A circular channel of radius a and length l connects a gas filled chamber at number density n_0 to a vacuum chamber. The gas which passes through the channel must flow around a circular disc of radius R arranged coaxially with the channel and spaced a distance S from the flat wall W. It is assumed that the gas pressure $P_0=n_0kT$ is determined by direct measurement with a suitable gauge.

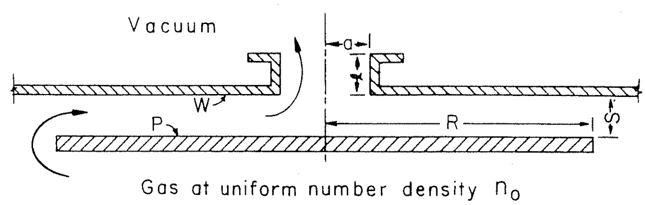

Figure 1. Arrangement of conductance elements illustrating how the cylindrical channel of radius a and length l is shielded from the gas source by circular disc P of radius R.

It is further assumed that the conductance channel is effectively screened from variations in the cell geometry beyond the circular disc and that the gas pressure in the cell is everywhere uniform and equal to P_0 up to the entrance to the cylindrical gap between the surface P of the disc and the parallel surface W of the cell wall. While it is true that this assumption merely moves the artificial plane of equilibrium from the circular channel entrance to a

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distant cylindrical ring, there are some decided advantages in doing so. First the ring source has less direct exposure to the vacuum sink and can, therefore, be more nearly at the equilibrium state. Since the ratio of the characteristic dimension l_c of an aperture to the gas mean free path λ determines the justification of using the undisturbed equilibrium approximation, it is evident that a given entrance area satisfies the condition ($l_c < \lambda$) better as a long narrow slit ($l_c = S$) than as a circle ($l_c = a$). A further advantage of the proposed configuration results from the fact that it may be possible to ease the experimental problem of measuring the low pressure P_0 accurately. This could be accomplished by using shielded channels having S and (or) $1/R$ sufficiently small with regard to channel radius a to establish a large but known density gradient between the entrance to the shield and the channel. Then for a given net flow through the channel the actual cell pressure at the gauge will be substantially larger and easier to measure than in the absence of the shield.

Calculations

The various methods for calculating the transmission of conductive elements in steady state effusive flow can be grouped in two categories: (1) iterative methods and (2) statistical methods. This is illustrated by the methods of refs 1-7. The statistical (Monte Carlo) methods are better suited for use with systems of complex geometry, but the iterative methods can yield smoother results and resolve finer detail when applied to systems of suitably simple geometry[8]. The present calculations were made by an iterative method developed here and described in previous reports[8-12].

As usual the surfaces defining the region of molecular flow were divided into a number of elements (concentric rings on surfaces W and P). The dimensions are all reduced in terms of the disc radius R; $R^* = R/R = 1$, etc. Each element is assumed to radiate gas molecules diffusely at a rate compatible with a uniform reduced radiative number density $n_i^* = n_i/n_0$. Initial trial values of n_i^* were assigned, and final steady state values were determined by a standard iterative procedure in which new values of n_i^* are given by

$$n_i^* = \sum_j n_j^* F_{ij} + F_i^0 \tag{1}$$

where F_{ij} and F_i^0 are geometric factors. F_{ij} relates the exposure of a median point on the receiving ring i to the radiating ring j (Figure 2) and is defined as

$$F_{ij} = 2S^* \int_{r_{j_1}^*}^{r_{j_2}^*} \int_0^{\pi} \frac{d\phi}{(S^{*2} + r_i^{*2} + r^{*2} - 2r_i^* r^* \cos\phi)^2} r^* \, dr^* \tag{2}$$

which reduces to

$$F_{ij} = \frac{1}{2} \left\{ \frac{r_j^{*2} - r_i^{*2} - S^{*2}}{[(r_j^{*2} + r_i^{*2} + S^{*2})^2 - 4r_i^{*2} r_j^{*2}]^{1/2}} \right\} \Bigg|_{r_{j_1}^*}^{r_{j_2}^*} . \tag{3}$$

F_i^0 similarly relates the exposure of the same median point on ring i to the cylindrical source (n^* source = 1) and is given by (Figure 3).

$$F_i^0 = \frac{1}{2} \left\{ 1 - \frac{1 - r_i^{*2} - S^{*2}}{[(1 - r_i^{*2} - S^{*2})^2 + 4S^{*2}]^{1/2}} \right\}. \tag{4}$$

Equations 1-4 are all that is necessary to determine the steady state radiative densities in the shielded region of channels with zero length l (i.e. for a knife edged circular orifice). When the channel has finite length, however, the problem is greatly complicated by the fact that points on the channel wall are not directly exposed to all of a radiative element on the shield if any of that element lies beyond $r^* = a^*$. The multiple integrals giving the necessary geometric factors F_{ij} in this case can not be reduced to closed form. Rather than attempt to compute these factors numerically an approximation was introduced to facilitate the calculation of transmission coefficients of shielded short tubes with aspect ratios $l/a > 0$. This was done by considering surface W (Figure 1) which contains the entrance orifice of the channel to be a continuous physical surface. Each ring on W located beyond the entrance orifice was considered as usual to be a perfect diffuse reflector (i.e. the reflection coefficient $\gamma = 1$). Those rings on W which lie in the entrance orifice, however, were assumed to be partial absorbers (i.e. $\gamma_i < 1$). The fraction γ of molecules crossing the orifice from the gas cell was assumed to be reflected back into the shield as if the orifice plane were a diffuse reflector. The fraction $1-\gamma$ of those molecules crossing the orifice eventually pass through the tube and flow into the pumped chamber. The latter fraction was taken to be the conventional Clausing factor f_C for the channel; therefore

$$\gamma_i = 1 - f_C \tag{5}$$

for rings on the entrance orifice and $\gamma_i = 1$ elsewhere. Equation 1 then becomes

$$n_i^* = \sum n_j^* \gamma_j F_{ij} + F_i^0. \tag{6}$$

The net flow rate Q through the channel reduced in terms of the conventional flow rate $n_0 \bar{v} \pi a^2/4$ is

$$Q^* = \frac{Q}{n_0 \bar{v} \pi a^2/4} = \frac{\sum_{i=1}^{n_S} \sum_j n_j^* F_{ij} A_i^* (1-\gamma_i)}{\pi a^{*2}} + \sum_{i=1}^{n_S} \frac{F_i^0 A_i^* (1-\gamma_i)}{\pi a^{*2}} \tag{7}$$

where in the first term the first sum is taken over the n_s rings comprising the entrance orifice and the second over all of the radiative elements on surface P, and A_i^* is the reduced area of the ith orifice ring. Finally the conductance C of the shielded channel is defined as

$$C = \frac{Q}{n_0} = Q^* \bar{v} \pi a^2/4 = Q^* C_0 \tag{8}$$

where $C_0 = \bar{v} \pi a^2/4$ is the idealized conductance of a circular orifice when the gas equilibrium inside the cell is not disturbed by the orifice. Equation (8) shows that Q^* is the transmission coefficient to be used in place of the Clausing factor. Its value depends on the geometry of the shield as well as the aspect ratio of the channel.

For a given shield size and channel orifice radius, the departure of Q^* from the Clausing factor will be the greatest for $l/a = 0$ and decrease as l/a increases. Values of Q^* were calculated for a number of channels for which $0.01 \le a^* = S^* \le 0.1$ and $0 \le l/a \le 5$. The entrance to the channel lying in the plane of surface W was divided into nine rings of uniform width. The remainder of surface W in line with surface P was divided into a number of uniform rings varying from 19 for $a^* = 0.1$ to 100 for $a^* = 0.01$. Surface P

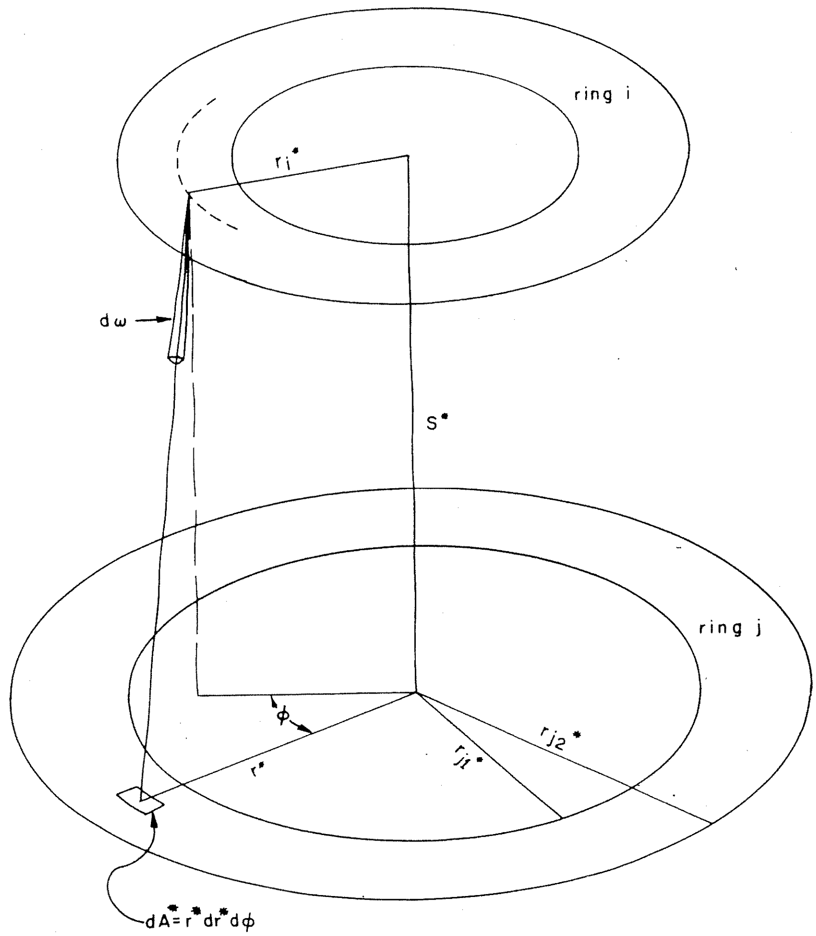

Figure 2. Coordinate system used to calculate the geometric factor F_{ij} related to the exposure of a median point on ring i to the radiating ring j.

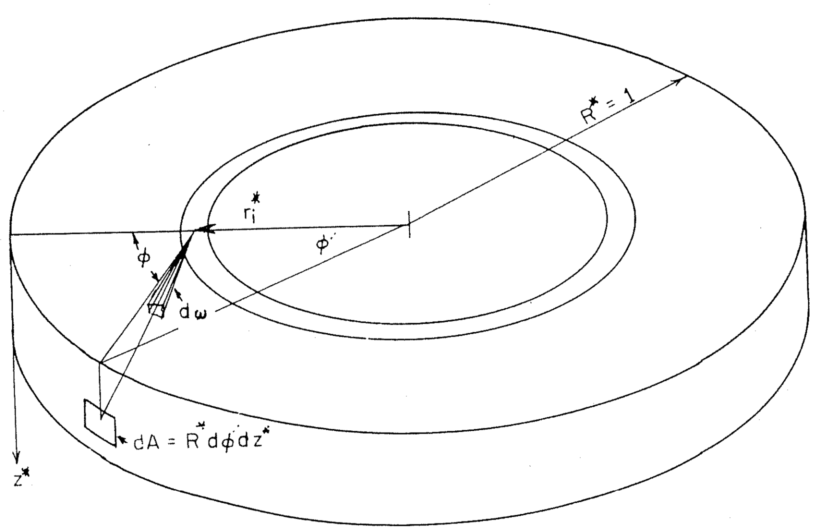

Figure 3. Coordinate system used to calculate the geometric factor F_i^0 related to the exposure of a median point on ring i to the cylindrical source.

was divided into nine rings within the central circle of radius a^* and the same number as on W beyond $r^*=a^*$. Selected runs were made with initial trial values of n_i^* both higher and lower than the steady state values to assure that the procedure yielded proper convergence.

Results

Table 1 lists the calculated values of the transmission coefficients Q^* for those configurations which were included in the present study. In each case the shield disc spacing S is equal to the channel radius a. The shield disc radius is equal to a/a^* which for the last five columns of Table 1 gives 100, 75, 50, 25 and 10 channel radii respectively. Values of the Clausing factor f_C are included for comparison. These depend only on the aspect ratio l/a.

Table 1. Calculated values of the transmission coefficient Q^* for circular channels of various aspect ratios l/a and reduced radii a^* shielded by circular discs of unit radius having spacing $S^*=a^*$. Clausing factors f_C are included for comparison

		Q^*				
l/a	$a^*=S^*$ / f_C	0.01	0.01333	0.02	0.04	0.10
0	1.000	0.5235	0.5333	0.5466	0.5713	0.6132
0.2	0.9092	0.4974	0.5061	0.5182	0.5404	0.5778
0.5	0.8013	0.4631	0.4707	0.4812	0.5004	0.5322
1	0.6720	0.4164	0.4228	0.4313	0.4467	0.4719
1.5	0.5810	0.3790	0.3847	0.3919	0.4045	0.4251
2	0.5136	0.3493	0.3539	0.3600	0.3707	0.3879
3	0.4205	0.3033	0.3069	0.3116	0.3196	0.3323
4	0.3589	0.2697	0.2726	0.2764	0.2827	0.2926
5	0.3146	0.2434	0.2462	0.2494	0.2545	0.2625

The nature of the steady state gradient of gas density in the shield region is illustrated for some typical cases in Figure 4. The reduced effective radiative number densities $n_P^*(r^*)$ are plotted vs reduced radial position r^* along the surface of the shield disc P for channels of zero aspect ratio (i.e. knife edge circular apertures) and various reduced aperture radii a^*. The corresponding gradient plots along surface W are similar except that at each radius $r^* \geq a^*$, $n_W^*(r^*) > n_P^*(r^*)$. This inequality results from the fact that points on surface W are less exposed to the sink than are corresponding points on surface P.

Discussion

A flowing gas cannot be at equilibrium, and strictly there are no regions in which the distribution is uniform and equal to that of an equilibrium system. The artificial device of dividing the system into two distinct regions separated by a boundary at which the density gradient abruptly sets in can at best lead to only approximately correct results. Such results may be useful, however, if the onset of the gradient is sufficiently gentle. For this reason conventional theoretical transmission coefficients such as Clausing's are more reliable as the aspect ratio l/a increases. For circular apertures or short cylindrical channels the Clausing factors are in substantial error and the magnitudes of those errors depend on the details of the geometry of the effusion cell. It is recommended then that those factors not be used for apertures or short tubes ($l/a \lesssim 10$).

When it is necessary or desirable to use short effusion channels a separate calculation for each particular cell geometry should be made, or each channel should be shielded from the bulk of the effusion cell in a way which produces a definable gradient with a gentle onset. The worst case considered here is that for which $S^*=a^*=0.1$ and $l/a=0$. The resulting gradient on the shield surface P is shown in Figure 4. The slope of the gradient as $r^* \rightarrow 1$ is 0.0015 and the value of $n_P^*(r^*=1)$ is 0.988 leading to a discontinuity across the dividing boundary of $\Delta n^*=0.012$. The corresponding values for the case $S^*=a^*=0.01$ and $l/a=0$ are: initial slope$=10^{-4}$ and $\Delta n^*=0$.

The shielding arrangement recommended here is one of many possibilities considered. It was selected for these calculations because of its mathematical simplicity and because it should not be difficult to fabricate effusion sources which closely resemble the

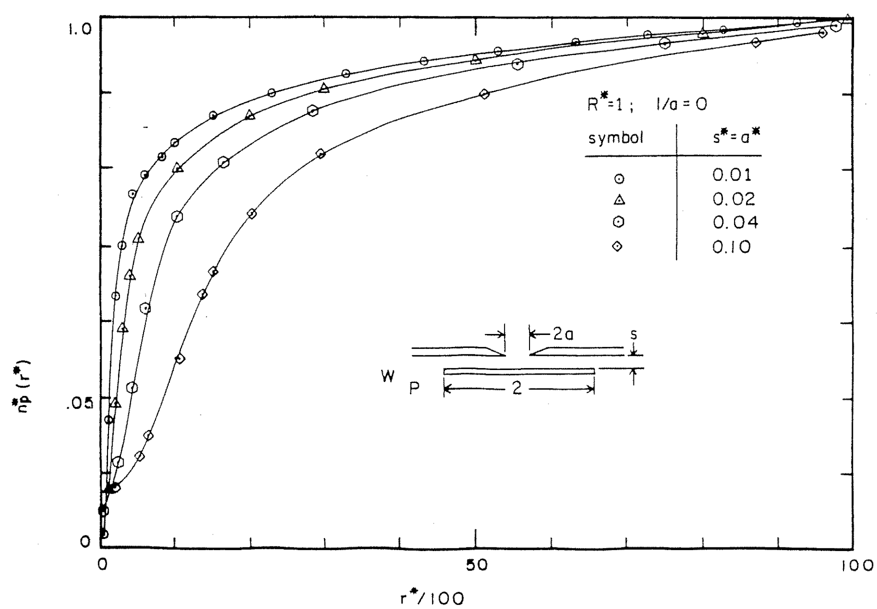

Figure 4. Reduced radiative density gradients on shield plate P for some shielded thin apertures.

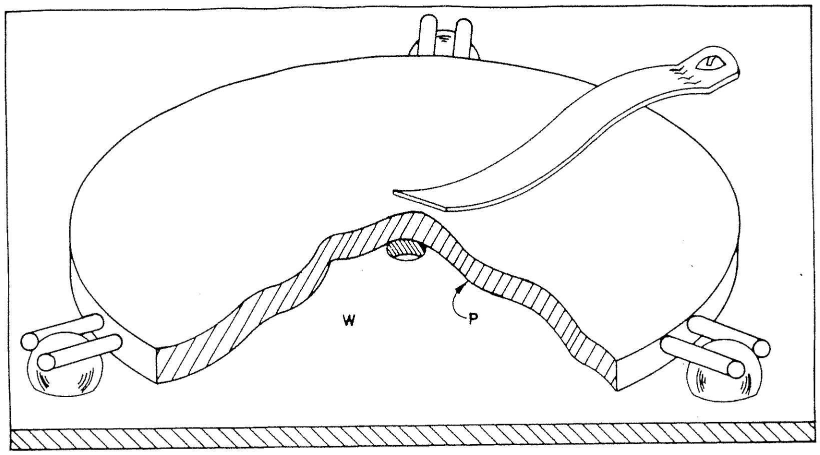

Figure 5. Kinematic design of a demountable aperture shield.

model. The experimentalist will see many obvious ways to accomplish this. One method which is quickly demountable and kinematically stable is illustrated schematically in Figure 5. Three rounded posts or balls are fixed to surface *W*. They may be press fitted, soldered, cemented or otherwise secured. Three pairs of round pins are pressed into holes reamed into the side wall of the shield disc. The disc is held in place by means of a suitable spring clamp using light force to avoid excessive deformation of the assembly.

Acknowledgements

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References

[1] P Clausing, *Physica*, **9**, 65 (1929).
[2] P Clausing, *Zeit F Physik*, **66**, 471 (1931).
[3] C E De Marcus, US Atomic Energy Comm Report K-1302, Parts I and II (1956); Parts III and IV (1957).
[4] S Dushman, *Scientific Foundations of Vacuum Technique*, 2 edn, Chapt 2. John Wiley & Sons Inc, NY (1962).
[5] S Dushman, The production and measurement of high vacua, *Gen Elec Rev* (1922).
[6] G N Patterson, *Kinetic Theory of Gas Flows*, Chap V. University of Toronto Press, Toronto (1971).
[7] G A Bird, *Molecular Dynamics*, Chapt 2. Clavender Press, Oxford (1976).
[8] B P Mathur and S O Colgate, *Phys Rev A*, **6**, 1266 (1972).
[9] R N Nelson and S O Colgate, *Phys Rev A*, **8**, 3045 (1973).
[10] B P Mathur, J E Field and S O Colgate, *Phys Rev A*, **11**, 830 (1975).
[11] S O Colgate, *Vacuum*, **21**, 483 (1971).
[12] S O Colgate and C B Smith, *Vacuum*, **23**, 313 (1973).